

A. Vyšetřete konečnost integrálů.

$$1. \int_0^{\infty} \frac{\sqrt[3]{x} + 3}{\sqrt{x} + 2} dx \quad 2. \int_0^1 \frac{\sin(\pi x)}{x^{9/8} \ln x} dx \quad 3. \int_0^{\infty} \frac{\ln(x+1)}{x^{101/100}} dx$$

$$*4. \int_0^{\infty} \frac{\sin^2 x}{x^a} dx \quad 5. \int_0^{\infty} \sin^3\left(\frac{1}{\sqrt{x}}\right) dx \quad 6. \int_0^{\pi} \frac{dx}{\sqrt{\sin x}}$$

$$7. \int_0^{\infty} x \exp(-x) \sin x dx \quad *8. \int_0^{\infty} \frac{x}{1 + x^5 \sin^2 x} dx \quad 9. \int_0^{1/2} \frac{dx}{x \ln^b x}$$

$$10. \int_0^{\infty} \frac{dx}{\exp x - \cos x} \quad 11. \int_0^{\infty} \frac{\operatorname{arctg} x}{x^a} dx \quad *12. \int_0^{\infty} \frac{p(x)}{q(x)} dx$$

B. Vyšetřete spojitost funkce  $F(a)$ :

$$1. \int_0^{\infty} \frac{\exp(-ax)}{1+x^3} dx, \quad a \geq 0 \quad 2. \int_0^{\infty} \frac{x}{1+x^a} dx, \quad a > 2$$

$$3. \int_0^{\pi} \frac{\sin x}{x^a(\pi-x)^a} dx, \quad a < 2 \quad 4. \int_{-1}^1 \sqrt{x^2+a^2}, \quad a \in \mathbb{R}$$

$$5. \int_0^{\infty} \exp(-ax) \frac{\sin x}{x} dx, \quad a > 0 \quad 6. \int_1^{\infty} \frac{\sin\left(\frac{1}{x}\right)}{x(a+x)^2}, \quad a > -1$$

C. Derivováním podle parametru spočtěte integrály:

$$1. \int_0^{\infty} \frac{1 - \exp(-ax^2)}{x^2 \exp(x^2)} dx \quad 2. \int_0^1 \frac{x^a - 1}{\ln x} dx$$

$$3. \int_0^{\infty} \exp(-x) \frac{\sin(ax)}{x} dx \quad 4. \int_0^{\infty} \exp(-x) \frac{1 - \cos(ax)}{x} dx$$

$$5. \int_0^{\infty} \frac{\operatorname{arctg}(ax)}{x(1+x^2)} dx \quad * 6. \int_0^{\infty} \exp\left\{-x^2 - \frac{a^2}{x^2}\right\} dx$$

$$7. \int_0^{\pi/2} \frac{\ln(1 + a \sin x)}{\sin x} dx \quad 8. \int_0^{\pi} \frac{\ln(1 + \cos x)}{\cos x} dx$$