(16) víme: $\mathcal{F}[e^{-\pi x^2}] = e^{-\pi \xi^2}$ *)

$$\mathcal{F}[f(cx)] = \frac{1}{c}\hat{f}\left(\frac{\xi}{c}\right)$$

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odtud: $\mathcal{F}\left[e^{-ax^2}\right] = \left(\frac{\pi}{a}\right)^{\frac{1}{2}} \exp\left(-\frac{\pi^2}{a}\xi^2\right)$

$$c = \left(\frac{a}{\pi}\right)^{1/2}.$$

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$$\| e^{-ax^2}\|_{L^2}^2 = \int_{\mathbb{R}} e^{-2ax^2}\,dx = \sqrt{\frac{\pi}{2a}}.$$

$$\| \widehat{e^{-ax^2}}\|_{L^2}^2 = \left\|\left(\frac{\pi}{a}\right)^{1/2} e^{-2\frac{\pi^2}{a}\xi^2}\right\|_{L^2}^2 = \frac{\pi}{a}\cdot\sqrt{\frac{\pi}{2\pi^2/a}} = \sqrt{\frac{\pi}{2a}}.$$

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Konvoluce: $e^{-ax^2} * e^{-bx^2} = g(x) = ?$

$$\mathcal{F}\left(g(x)\right) = \widehat{e^{-ax^2}}\cdot\widehat{e^{-bx^2}} = \frac{\pi}{\sqrt{ab}}\cdot\exp\left(-\pi^2\xi^2\left(\frac{1}{a}+\frac{1}{b}\right)\right).$$

$$g(x) = \mathcal{F}_{-1}(\quad) = \frac{\pi}{\sqrt{ab}}\cdot\left(\frac{\pi}{c}\right)^{1/2}\cdot\exp\left(-\frac{\pi^2}{c}x^2\right)$$

$$c = \pi^2\left(\frac{1}{a}+\frac{1}{b}\right).$$

srovnej: $\mathcal{F}^{-1}\left(e^{-a\xi^2}\right) = \left(\frac{\pi}{a}\right)^{1/2} \exp\left(-\frac{\pi^2}{a}x^2\right)$

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*) při značení $\mathcal{F}[f](\xi) = \int_{R} f(x)\, e^{-2\pi i \xi x}\,dx$