Besselova rce : [určení řešení] $\quad x^2y''+xy'+(x^2-n^2)y=0 \quad n\in R$.

$$y''+\frac{1}{x}y'+\left(1-\frac{n^2}{x^2}\right)y=0$$

$$y(x)=\sum_{k=0}^{\infty}a_k x^{\rho+k}\,;\quad \rho\in R \quad \text{(určíme později)}$$

$$y''=\sum_{k=0}^{\infty}a_k(\rho+k)(\rho+k-1)\,x^{\rho+k-2}$$

$$y'=\sum_{k=0}a_k(\rho+k)\,x^{\rho+k-1}$$

$$\sum_{k=0}^{\infty}a_k(\rho+k)(\rho+k-1)\,x^{\rho+k}+a_k(\rho+k)\,x^{\rho+k}-n^2a_k x^{\rho+k}$$
$$+\sum_{k=0}^{\infty}a_k x^{\rho+k+2}$$

$$\sum_{k=2}^{\infty}a_{k-2}\,x^{\rho+k}$$

$$\sum_{k=2}^{\infty}x^{\rho+k}\left[a_k\left((\rho+k)^2-n^2\right)+a_{k-2}\right]$$

$$a_0=a_1=0 \qquad k=0:\ x^{\rho}a_0(\rho^2-n^2)+\quad +x^{\rho+1}a_1$$

$$(\rho+k)^2-m^2=\cancel{\rho^2}+2\rho k+k^2\cancel{-n^2}=2nk+k^2=k(k+2n).$$

$\boxed{\rho=n}$

$a_0=a_1=0$

a_2 … libovolné;
$$a_k=\frac{-a_{k-2}}{k(k+2n)}$$

$a_1=0:\ a_{2k+1}=0.$

$$a_{2k}=-\frac{a_{2k-2}}{2k(2k+2n)}=-\frac{a_{2(k-1)}}{4k(k+n)}$$

$\therefore\ b_k = a_{2k}$: $b_k = -\dfrac{b_{k-1}}{4\cancel{k}(k+\nu)}$

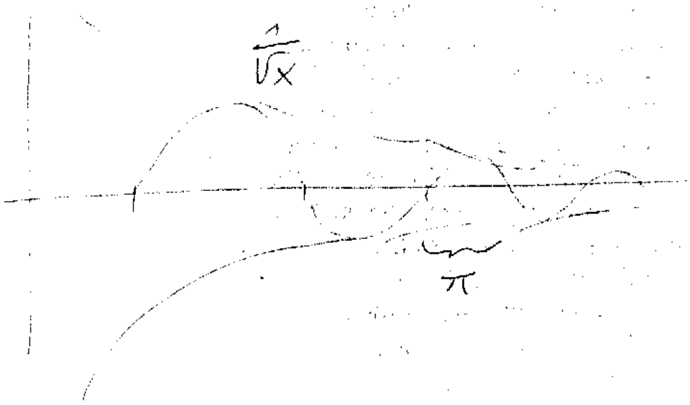
$$a_{2k} = (-1)^k \cdot \frac{1}{4^k} \cdot \frac{1}{k!} \cdot \frac{1}{(k+\nu)(k-1+\nu)} \cdots \frac{a_0}{(1+\nu)}$$

$$a_0 = b_0 = \frac{1}{2^\nu \Gamma(\nu+1)}\ ;$$

$\Gamma(\nu+1)(\nu+1) = \Gamma(\nu+2)$: $b_k = \dfrac{(-1)^k}{2^{2k+\nu} \cdot k!\, \Gamma(k+\nu+1)}$

řeš: $$J_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma(\nu+k+1)} \left(\frac{x}{2}\right)^{2k}.$$

Besselova fce 1. druhu o indexu $\underline{\nu}$.



$$J_\nu(x) \sim \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{2}\left(\nu + \frac{1}{2}\right)\right).$$

pro $x \gg \nu$