

(30)

$$Hf = f * \text{v.p.} \frac{1}{x}$$

$$\begin{aligned} \|Hf\|_{L^2} &= \|\widehat{Hf}\|_{L^2} = \|\widehat{f} \cdot \widehat{\text{v.p.} \frac{1}{x}}\|_{L^2} = \|\widehat{f} \cdot (-i\pi \operatorname{sgn} \xi)\|_{L^2} \\ &= \pi \|\widehat{f}\|_{L^2} \end{aligned}$$

$$\widehat{\text{v.p.} \frac{1}{x}} = -i\pi \operatorname{sgn}(\xi)$$

arican, nelo Čiňák s. 46; Pr 2.33

Jde o power law rozruš: $\|g\|_{L^2} = \|\widehat{g}\|_{L^2}$ (Parseval)

$$\widehat{f * g} = \widehat{f} \cdot \widehat{g}$$

a modného porovnaní:

$$\|\widehat{f}(-i\pi \operatorname{sgn}(\xi))\|_{L^2}^2 = \int |\widehat{f}(\xi) \cdot (-\pi i \operatorname{sgn}(\xi))|^2 d\xi$$

$$= \int |\widehat{f}(\xi)|^2 \pi^2 d\xi = \pi^2 \|\widehat{f}\|_{L^2}^2$$

neboť $|\operatorname{sgn}(\xi)| = 1$ skoro všude.