Vedení tepla na kruhu $\quad \partial_t u - \Delta u = 0 \quad ; (x_1,x_2) \in \Omega = \{x_1^2+x_2^2 < a^2\}$

$u = u(x,y,t)$ $\qquad u|_{r=a} = 0$

$x_1 = r\cos\varphi$ $\qquad u|_{t=0} = f.$

$x_2 = r\cdot\sin\varphi$

(a) radiální symetrie $\qquad f = f(r)$

$$u = u(r,t).$$

$$\Delta u = \partial_{rr}u + \frac{1}{r}\partial_r u + \frac{1}{r^2}\partial_{\varphi\varphi}u.$$

$$\partial_t u - \left(\partial_{rr}u + \frac{1}{r}\partial_r u\right) = 0.$$

Fourierova metoda: $\quad u(r,t) = R(r)\cdot c(t).$

$$c'(t)\,R(r) - c(t)\left[R''(r) + \frac{1}{r}R'(r)\right] = 0$$

$$\frac{c'(t)}{c(t)} = \frac{R''(r) + \frac{1}{r}R'(r)}{R(r)} = -\lambda^2 \quad \ldots \lambda \text{ -- určíme později}$$

(= ok. podm...)

1rce.

1rce: $\quad R'' + \frac{1}{r}R' + \lambda^2 R = 0 \quad \ldots$ Besselova rce:

obecná rce: $\quad R'' + \frac{1}{r}R' + \left(1 - \frac{m^2}{r^2}\right)R = 0 \qquad m = 0$

(B_0): $\quad r^2 R'' + r R' + (r^2 - 0^2) R = 0 \quad ;/ : r > 0$

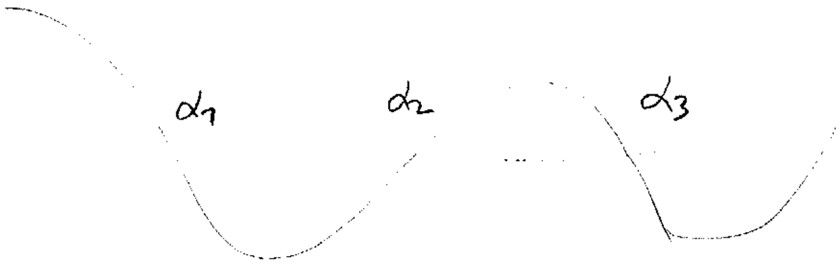
$$R'' + \frac{1}{r} R' + R = 0$$

$J_0(r) \cdots$ řešení ... $\Rightarrow$

$R(r) = J_0(\lambda r)$ řeš. $R'' + \frac{1}{r} R' + \lambda^2 R = 0$. [d. ú.]

? volba λ: $\quad R(a) = J_0(\lambda a) = 0 \cdots \quad \lambda a = \alpha_m$

$$\lambda = \lambda_m := \frac{\alpha_m}{a}$$



α_m; $m = 1, 2, \dots$ kořeny $J_0(r)$.

$$u_m(r) = J_0\Big(\overbrace{\frac{\alpha_m}{a}}^{\lambda_m} \cdot r\Big); \quad \text{řeš. } (2) \text{ pro } \lambda = \lambda_m; \quad u_m(a) = 0.$$

(1) $\quad \frac{c'(t)}{c(t)} = -\lambda_m^2; \quad c(t) = e^{-\lambda_m^2 t} \cdot C_m$

obecné řešení: $\quad u(r,t) = \sum_{m=1}^{\infty} C_m e^{-\lambda_m t} \cdot u_m(r)$.

$C_m = ?$ počáteční podmínka:

$$u(r,0) = \sum_{m=1}^{\infty} C_m u_m(r) = f(r).$$

Tvrzení: $\{u_m\}$ jsou OG na $(0,a)$ s vahou r.

tj.: $\langle u_m, u_n \rangle = \int_0^a u_m(r) u_n(r)\, r\, dr = 0 \quad ; m \neq n.$

$$u_m'' + \frac{1}{r} u_m' + \lambda_m u_m = 0 \qquad / \; r$$

$$r u_m'' + u_m' + \lambda_m u_m \cdot r = 0$$

$$(r u_m')' + \lambda_m^2 u_m \cdot r = 0 \qquad ; \quad u_n$$

$$(r u_n')' + \lambda_n^2 u_n r = 0 \qquad ; \quad \cancel{u_m u_m = w} \; u_m$$

$$(r w')' +$$

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obecné věta: $\quad (a(t) u')' + \lambda b(t) u = 0 \qquad , \; v$

$\;\;$a. geneřtor.. $\quad (a(t) v')' + \tilde{\lambda} b(t) v = 0 \qquad u$

$$a_m = \frac{\langle u_m, f \rangle}{\langle u_m, u_m \rangle}.$$