B.. rovnice: $u = u(r, \varphi, t)$

Ansatz: $u = R(r) \cdot e^{im\varphi} \cdot T(t)$

$$\partial_t u = \partial_{rr} u + \frac{1}{r}\partial_r u + \frac{1}{r^2}\partial_{\varphi\varphi} u \quad ;$$

$$T' \cdot R \cdot \cancel{e^{im\varphi}} = \left[R'' + r^{-1} R' + (r^{-2} \cdot (-m^2)) R \right] T \cdot 1 \cdot \cancel{e^{im\varphi}}$$

$$\frac{T'}{T} \underset{(1)}{=} \frac{R'' + r^{-1} R' - m^2 r^{-2} R}{R} \underset{(2)}{=} -\lambda^2$$

$$\boxed{\begin{array}{l}\text{aside:} \\ -\Delta u = \lambda^2 u \\ u|_{\partial\Omega} = 0\end{array}}$$

$$(2): \quad R'' + r^{-1} R' + \left(\lambda^2 - \frac{m^2}{r^2}\right) R = 0 \quad , \quad u_{n,m}(r) = J_m(\lambda_{n,m} r)$$

subst.: $\boxed{\lambda r = \rho}$

$$(B_m) \quad Z'' + \rho^{-1} Z' + \left(1 - \frac{m^2}{\rho^2}\right) Z = 0 \qquad Z = Z(\rho).$$

$$J_m(\rho) = \left(\frac{\rho}{2}\right)^m \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,(n+m)!} \left(\frac{\rho}{2}\right)^{2n}$$

kde $J_m(\lambda_{n,m} a) = 0$

tj: $\lambda_{n,m} = \frac{\alpha_{n,m}}{a}$

$\{\alpha_{n,m}\}_{n=1}^{\infty}$ .. kořeny $J_m(\rho)$.

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obecné řešení: $\sum_{n,m} c_{n,m} \, e^{-\lambda_{n,m}^2 t} \cdot e^{im\varphi} \cdot u_{n,m}(r)$.

poč. podm.: $\sum c_{n,m}\, e^{im\varphi}\, u_{n,m}(r) = u_0(r,\varphi)$

<u>**leč:**</u> $u_0(r,\varphi) = \sum_{m\in\mathbb{Z}} d_m(r)\, e^{im\varphi}$

↑ rozvoj do Four. řady

$\forall m$ pevné: $\sum_n c_{n,m}\, u_{n,m}(r) = d_m(r)$

↑ $L^2_r(0,a)$..

OG v $(0,a)$ s vahou $r\,dr$

$$c_{n,m} = \frac{\langle u_{n,m}(r), d_m(r)\rangle}{\langle u_{n,m}, u_{n,m}\rangle}.$$

<u>rozvoj:</u> $v(x) = \sum_k c_k\, u_k(x)$

u_k jsou OG vůči $\langle\cdot,\cdot\rangle$.

$$\Rightarrow\quad c_k = \frac{\langle v, u_k\rangle}{\langle u_k, u_k\rangle}.$$
