

$$\textcircled{1} f(x, y) = x^3 - 9x^2 + 15x - y^2 + 2y$$

$$\nabla f = 0 : \quad A = (1, 1) \quad \nabla^2 f = \begin{pmatrix} 6x - 18 & 0 \\ 0 & -2 \end{pmatrix}$$

$$B = (5, 1)$$

$$\nabla^2 f(A) = \begin{pmatrix} -12 & 0 \\ 0 & -2 \end{pmatrix}; \quad \sigma = \{-12, -2\} \text{ -- lok. max.}$$

$$\nabla^2 f(B) = \begin{pmatrix} 12 & 0 \\ 0 & -2 \end{pmatrix}; \quad \sigma = \{12, -2\} \text{ -- sedlový bod.}$$

$$? \text{ globální: } \varphi(t) := f(t, 0) = t^3 - 9t^2 + 15t$$

$$\rightarrow \pm \infty \text{ pro } t \rightarrow \pm \infty, \quad = t^3 \left(1 - \frac{9}{t} + \frac{15}{t^2}\right)$$

$\Rightarrow f$ nemá žádné mezerové

\nexists globální extrém.

$$\textcircled{2} f(x, y) = y^3 + \frac{3}{2}y^2 - 18y + x - x^2$$

$$\nabla f = 0 : \quad A = \left(\frac{7}{2}, -3\right) \quad \nabla^2 f = \begin{pmatrix} -2 & 0 \\ 0 & 6y + 3 \end{pmatrix}$$

$$B = \left(\frac{7}{2}, 2\right)$$

$$\nabla^2 f(A) = \begin{pmatrix} -2 & 0 \\ 0 & -15 \end{pmatrix} : \text{ lok. max.}$$

$$\nabla^2 f(B) = \begin{pmatrix} -2 & 0 \\ 0 & 15 \end{pmatrix} : \text{ sedlový bod}$$

\nexists globální extrém \nexists

$$\varphi(t) = f(0, t) = t^3 + \frac{3}{2}t^2 - 18t$$

je mezerové šance i zde...

$$\textcircled{3} \quad f(x, y) = xy + x^2 + y^2 - 3x - 4y$$

$$\nabla f = 0: \quad A = \left(\frac{2}{3}, \frac{5}{3} \right); \quad \nabla^2 f = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\Delta(\nabla^2 f(A)) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}; \quad \sigma = \{3, 1\} \dots \text{lok. min.}$$

Trvdme: $\lim_{x^2+y^2 \rightarrow \infty} f(x, y) = +\infty.$

polární souřadice: $x = r_m \cos \varphi_m$; $r_m \rightarrow +\infty$
 $y = r_m \sin \varphi_m$ $\{\varphi_m\}$ libovolné.

$$\begin{aligned} f &= r_m^2 (1 + \cos \varphi_m \sin \varphi_m) - 3r_m \cos \varphi_m - 4r_m \sin \varphi_m \\ &= r_m^2 \left(1 + \underbrace{\frac{1}{2} \sin 2\varphi_m}_{> -\frac{1}{2}} - \underbrace{\frac{3}{r_m} \cos \varphi_m - \frac{4}{r_m} \sin \varphi_m}_{\rightarrow 0} \right) \end{aligned}$$

$$\geq r_m^2 \left(\frac{1}{2} + o(1) \right) \rightarrow +\infty ; m \rightarrow \infty.$$

Vše z předchozí: \exists globální minimum

A jediný lokální bod;

vyso: A je glob. min.

$$\textcircled{4} \quad f(x, y) = -x^2 - 2y^2 + 2xy - x - 2y$$

$$\nabla f = 0: \quad A = \left(-2, -\frac{3}{2} \right); \quad \nabla^2 f = \begin{pmatrix} -2 & 2 \\ 2 & -4 \end{pmatrix};$$

$$\sigma(\nabla^2 f(A)) = -3 \pm \sqrt{5} > 0 \dots \text{lok. max.}$$

$$\lim_{(x,y) \rightarrow \infty} f(x, y) = -\infty$$

\Rightarrow A je glob. max.

(podobně jako výše.)

$$\textcircled{5} \quad f(x, y, z) = x^2 + y^2 - z^2 + xy - yz + xz$$

$$\nabla f = 0: \quad A = (0, 0, 0); \quad \nabla^2 f = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & -2 \end{pmatrix}$$

$$\sigma(\nabla^2 f(A)) = \left\{ 3, \frac{-1 \pm \sqrt{17}}{2} \right\}$$

sedlový bod: funkce nemá žádné extrémny.

$$\textcircled{6} \quad f(x, y, z) = x^2 + y^2 + z^2 + 2xy - 2yz - 2xz$$

$$\nabla f = 0 \Leftrightarrow x + y - z = 0 \quad (\text{přirovina dimenze 2})$$

$$\nabla^2 f = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 2 \end{pmatrix}; \quad \sigma = \{0, 6\} \quad \text{-- indetinitní}$$

?! pozoruj: $f = (x + y - z)^2 \geq 0$

$$M = \{(x, y, z) \in \mathbb{R}^3; \dots x + y - z = 0\}$$

jsou všechno globální miníma.

$$\textcircled{7} \quad f(x, y) = (2x + y) e^x (x - y)$$

$$\nabla f = 0: \quad \begin{aligned} (y + 2x) + 2 &= 0 && \text{nemá řešení.} \\ -(y + 2x) + 1 &= 0 && \nexists \text{ extrémny} \end{aligned}$$

$$\textcircled{8} \quad f(x, y) = (2 + x - y) e^x (-x^2 - y^2)$$

$$\nabla f = 0: \quad \begin{aligned} (2 + x - y) \cdot (-2x) + 1 &= 0 \\ (2 + x - y) \cdot (-2y) - 1 &= 0 \end{aligned}$$

$$\text{Řešením: } (2 + x - y) \cdot (x + y) = 0$$

(a) $2 + x - y = 0$: nevyhoví původní vci

(b) $x = -y$:

$$1. \text{ ncl: } (2+2x)(-2x)+1=0$$

$$x = \frac{1}{2}(-1 \pm \sqrt{2}).$$

$$\rightarrow \text{noderself body: } A = \left(\frac{1}{2}(-1+\sqrt{2}), \frac{1}{2}(7-\sqrt{2})\right)$$

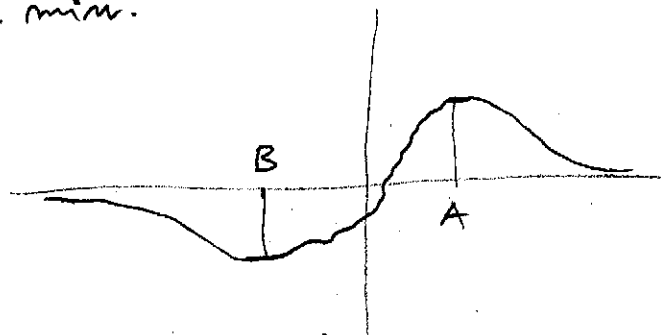
$$B = \left(\frac{1}{2}(-1-\sqrt{2}), \frac{1}{2}(7+\sqrt{2})\right).$$

$$\frac{D^2 f = \dots}{\dots}$$

$$\text{lec: } x^2+y^2 \rightarrow \infty: \underbrace{(2+x-y)}_{\text{polynom}} \cdot \underbrace{e^{-x}(-x^2-y^2)}_{\rightarrow 0 \text{ velmi rychle}} \rightarrow 0$$

$$f(A) > 0: \text{ glob. max.}$$

$$f(B) < 0: \text{ glob. min.}$$



$$\textcircled{9} \quad f(x,y) = (x+y) \cdot e^{-x} (x^2+2y^2)$$

$$Df=0: \quad (x+y) \cdot 2x + 1 = 0$$

$$(x+y) \cdot 4y + 1 = 0$$

$$\text{rozděl: } (x+y)(x-2y) = 0.$$

$$(a): x = -y: \text{ neřešit 1. ncl}$$

$$(b): x = 2y: \quad \underline{3y = 4y + 1 = 0}$$

20. nemá řešení!

✱ euklidy.