(24) $\int x^2 \sin^2 x \, dx = \int x^2 \left(\frac{1-\cos 2x}{2}\right) dx$

$\sin^2 y = \frac{1}{2}(1-\cos 2y)$

$\cos^2 y = \frac{1}{2}(1+\cos 2y)$

$$= \underbrace{\int \frac{x^2}{2} dx}_{\frac{x^3}{6}} + \underbrace{\int x^2 \left(-\frac{\cos 2x}{2}\right) dx}_{I_1}$$

$$I_1 = \int \underbrace{\frac{x^2}{2}}_{u} \underbrace{(-\cos 2x)}_{v'} dx = -\frac{x^2}{4}\sin 2x + \frac{1}{2}\underbrace{\int x \sin 2x \, dx}_{I_2}$$

$u' = x \; ; \; v = -\frac{\sin 2x}{2}$

$$I_2 = \int \underbrace{x}_{u} \underbrace{\sin 2x}_{v'} \, dx = -\frac{x}{2}\cos 2x + \frac{1}{2}\underbrace{\int \cos 2x \, dx}_{\frac{\sin 2x}{2}}$$

$u' = 1 \; , \; v = -\frac{1}{2}\cos 2x$

celkem: $\int x^2 \sin^2 x \, dx = \frac{x^3}{6} - \frac{x^2}{4}\sin 2x - \frac{x}{4}\cos 2x + \frac{\cos 2x}{8}$

v $\mathbb{R}$. [m.x.]

(25) $$\int \underbrace{x}_{u} \cdot \underbrace{\frac{1}{\cos^2 x}}_{v'} \, dx = x \cdot \operatorname{tg} x - \underbrace{\int \operatorname{tg} x \, dx}_{I_1}.$$

$u' = 1$; $v = \operatorname{tg} x$

$$I_1 = \int \frac{\sin x}{\cos x} dx \left| \begin{array}{l} y = \cos x \\ dy = -\sin x \, dx \end{array} \right\} = -\int \frac{dy}{y} = -\ln|y| = -\ln|\cos x|$$

kde? : v intervalech, kde $y \neq 0$ tj: $\left(-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi\right)$.

$k \in \mathbb{Z}$

$$\int \frac{x}{\cos^2 x} dx = x \operatorname{tg} x + \ln|\cos x| \text{ ; v intervalech}$$

[m.x.]

(26)

$$\int \underbrace{2 \cdot \operatorname{arctg} \frac{x}{2}}_{v} dx = 2x \operatorname{arctg} \frac{x}{2} - \underbrace{\int \frac{x}{1+\left(\frac{x}{2}\right)^2} dx}_{I_1}$$

$u' = 1$ $\quad v$

$u = x$ $\quad v' = \frac{1}{1+\left(\frac{x}{2}\right)^2}$

$$I_1 = 2\int \frac{x/2}{1+(x/2)^2} dx \left| \begin{array}{l} (x/2)^2 + 1 = y \\ x/2 \, dx = dy \end{array} \right\} = 2\int \frac{dy}{y}$$

$$= 2 \ln|y| = 2 \ln\left(1+(x/2)^2\right) \text{ v } \mathbb{R}.$$

celkem

$$2x \operatorname{arctg} \frac{x}{2} - 2 \ln\left(1 + \frac{x^2}{4}\right) \text{ v } \mathbb{R}.$$

[m.x.]

(27) $\int \underbrace{\arcsin x}_{v} \, dx = x \cdot \arcsin x - \underbrace{\int \frac{x}{\sqrt{1-x^2}} dx}_{I_1}$

$u' = 1 \quad v$

$u = x \quad v' = \frac{1}{\sqrt{1-x^2}}$

$I_1 = \frac{1}{2}\int \frac{2x}{\sqrt{1-x^2}} dx \; \left| \begin{array}{l} y = 1-x^2 \\ dy = -2x\,dx \end{array} \right\} = -\int \frac{dy}{2\sqrt{y}} = -\sqrt{y}$

$= -\sqrt{1-x^2}$

kde? v intervalech, kde $y > 0$; tj. $x \in (-1,1)$.

celkem: $\int \arcsin x \, dx = x \cdot \arcsin x + \sqrt{1-x^2}$ v $(-1,1)$.

[m. x.]

(28) $\int \underbrace{\frac{1}{x^2}}_{u'} \cdot \underbrace{\arccos x}_{v} \, dx = -\frac{\arccos x}{x} - \underbrace{\int \frac{dx}{x\sqrt{1-x^2}}}_{I_1}$.

$u = -\frac{1}{x} \quad v' = \frac{-1}{\sqrt{1-x^2}}$

$I_1 = \frac{1}{2}\int \frac{-2x\,dx}{x^2\sqrt{1-x^2}} \; \left| \begin{array}{l} y = 1-x^2 \\ dy = -2x\,dx \end{array} \right\} = \frac{1}{2}\int \frac{dy}{(1-y)\sqrt{y}} \; \left| \begin{array}{l} \sqrt{y} = t \\ y = t^2 \\ dy = 2t\,dt \end{array} \right.$

$= \int \frac{t\,dt}{(1-t^2)\cdot t} = \int \frac{dt}{1-t^2} = \frac{1}{2}\int \frac{1}{1-t} + \frac{1}{1+t} \, dt$

$= \frac{1}{2} \ln \left| \frac{1+t}{1-t} \right|$.

---

celkem: $\dots = -\frac{\arccos x}{x^2} + \frac{1}{2} \ln \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}$;

kde?: $x \in (-1,0)$

$x \in (0,1)$.

(29) $\int \underbrace{\ln\left(\sqrt{x+1}-\sqrt{x-1}\right)} dx$; $\mathcal{D} = ?$ $\quad x+1>0$ & $x-1>0$

$\boxed{x>1}$

$\sqrt{x+1} > \sqrt{x-1}$ o.k.

$u' = 1 \qquad v$

$$u = x \qquad v' = \frac{\frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x-1}}}{\sqrt{x+1}-\sqrt{x-1}} = \frac{1}{2\sqrt{x^2-1}} \cdot \frac{\sqrt{x-1}-\sqrt{x+1}}{\sqrt{x+1}-\sqrt{x-1}}$$

$$= -\frac{1}{2\sqrt{x^2-1}} \quad [\text{g.m.}]$$

per-partes: $\quad x \cdot \ln\left(\sqrt{x+1}-\sqrt{x-1}\right) + \underbrace{\int \frac{x}{2\sqrt{x^2-1}}dx}_{I_1}$

$$I_1 = \frac{1}{4}\int \frac{2x\,dx}{\sqrt{x^2-1}} \left| \begin{array}{l} y = x^2-1 \\ dy = 2x\,dx \end{array} \right\} = \frac{1}{4}\int \frac{dy}{\sqrt{y}} = \frac{1}{2}\sqrt{y}$$

$$= \frac{1}{2}\sqrt{x^2-1} \ .(\text{m.r.})$$

celem:

$$= x \cdot \ln\left(\sqrt{x+1}-\sqrt{x-1}\right) + \frac{1}{2}\sqrt{x^2-1} \ , \ v\ (1, +\infty).$$

(30) $$\int \frac{\ln \operatorname{arctg} x}{1+x^2} dx \left| \begin{array}{l} y = \operatorname{arctg} x \\ dy = \frac{dx}{1+x^2} \end{array} \right\} = \int \ln y \, dy$$

Per-partes: $\int \underbrace{1}_{u'} \cdot \underbrace{\ln y}_{v} \, dy = y \ln y - \int dy = y(\ln y - 1)$.

$v\ (0, +\infty)$.

$u = y \quad v' = \frac{1}{y}$

celem: $\quad = \operatorname{arctg} x \cdot \left(\ln \operatorname{arctg} x - 1\right)$; $x \in (0, +\infty)$.

[m.r.]

(31) $\displaystyle \int \frac{\operatorname{tg}(1/x)}{x^2}\,dx \;\left|\; \begin{aligned} y &= \tfrac{1}{x} \\ dy &= -\tfrac{1}{x^2}dx \end{aligned} \right. = -\int \operatorname{tg} y\, dy$

$\displaystyle = \int \frac{-\sin y\, dy}{\cos y} \;\left|\; \begin{aligned} z &= \cos y \\ dz &= -\sin y\, dy \end{aligned} \right. = \int \frac{dz}{z} = \ln|z|\,;$

kde platí? $z \in (-\infty, 0)$ nebo $(0, +\infty)$

$\cos y \neq 0: \quad y \in \left(-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi\right); \; k \in \mathbb{Z}$

$x \in \left(-\frac{2}{\pi}, 0\right)$ nebo $\left(0, \frac{2}{\pi}\right)$

$= \ln\left|\cos \frac{1}{x}\right|$ nebo $\left(\left(-\frac{\pi}{2} + k\pi\right)^{-1}, \left(\frac{\pi}{2} + k\pi\right)^{-1}\right), \; k \in \mathbb{Z} \setminus \{0\}.$

(32) $\displaystyle \int \frac{dx}{\cos^3 x} = \int \frac{\cos x\, dx}{\cos^4 x} \;\left|\; \begin{aligned} y &= \sin x\,; \quad \cos^2 x = 1 - y^2 \\ dy &= \cos x\, dx \end{aligned} \right.$

$\displaystyle = \int \frac{dy}{(1-y^2)^2} = I.$ $\quad y \in (-\infty, -1), (-1, 1), (1, +\infty).$

také: $\displaystyle J = \int \frac{dy}{1-y^2} = \frac{1}{2} \int \frac{1}{1-y} + \frac{1}{1+y}\, dy = \frac{1}{2} \ln\left|\frac{1+y}{1-y}\right|.$

per-partes: $\displaystyle J = \int 1 \cdot \frac{1}{1-y^2}\, dy = \frac{y}{1-y^2} - 2\int \frac{y^2}{(1-y^2)^2} = \frac{y}{1-y^2} + 2(J - I)$

$\displaystyle 2I = \frac{y}{1-y^2} + J\,;$

$\displaystyle I = \frac{+y}{2(1-y^2)} + \frac{1}{4} \ln\left|\frac{1+y}{1-y}\right|\,;$ y výše.

celkem: $\displaystyle = \frac{\sin x}{2\cos^2 x} + \frac{1}{4} \ln\left|\frac{1+\sin x}{1-\sin x}\right|$ $\quad x \in \left(-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi\right)$, $k \in \mathbb{Z}$.

(33) $\int \frac{x^2\,dx}{(8x^3+27)^{2/3}}$ $\quad \mathcal{D} = ?$ $\quad 8x^3 > -27$

$x^3 > (3/2)^3$

$x \in (3/2, +\infty)$.

$y = 8x^3 + 27$

$dy = 24x^2\,dx$ $\quad = \frac{1}{24}\int \frac{dy}{(y)^{2/3}} = \frac{1}{24}\int y^{-2/3}\,dy$

$= \frac{1}{24}\cdot 3\, y^{1/3} = \frac{1}{8}\left(8x^3+27\right)^{1/3}$ [m.x.]

(34) $\int \frac{\sin x + \cos x}{\sqrt[3]{\sin x - \cos x}}\,dx$ $\left|\begin{array}{l} y = \sin x - \cos x \\ dy = \cos x + \sin x\,dx \end{array}\right\} = \int \frac{dy}{\sqrt[3]{y}}$

$= \frac{3}{2}\sqrt[3]{y^2}$; $\quad y \in (-\infty, 0)$ nebo $(0, +\infty)$.

$\sin x = \cos x : \ \operatorname{tg} x = 1 : \ x = \frac{\pi}{4}$.

celkem: $= \frac{3}{2}\sqrt[3]{(\sin x - \cos x)^2}$; $\quad x \in \left(\frac{\pi}{4} + k\pi, \frac{5\pi}{4} + k\pi\right)$.

(35) $\int \frac{dx}{\sqrt{e^x - 1}}$ $\left|\begin{array}{l} y = \sqrt{e^x-1} \\ y^2 = e^x - 1 \\ x = \ln(y^2+1) \\ dx = \frac{2y}{y^2+1}\,dy \end{array}\right\} = \int \frac{2\cancel{y}\,dy}{\cancel{y}(y^2+1)}$

$x \in (0, +\infty)$.

$= 2\operatorname{arctg} y$.

$y \in \mathbb{R}$.

celkem:

$= 2\operatorname{arctg}\sqrt{e^x-1}$; $\quad x \in (0, +\infty)$.

M.X.