(45) $I_n = \int x^n e^{ax}$; $n \geq 0$ celé; $a \in \mathbb{R} \setminus \{0\}$.

$$I_{n+1} = \int \underbrace{x^{n+1}}_{u} \underbrace{e^{ax}}_{v'} dx = \frac{1}{a} x^{n+1} e^{ax} - (n+1) \int x^n \frac{1}{a} e^{ax} dx$$

$u' = (n+1) x^n$; $v = \frac{1}{a} e^{ax}$;

$$\boxed{I_{n+1} = \frac{x^{n+1} e^{ax}}{a} + \frac{n+1}{a} I_n \qquad I_0 = \int e^{ax} dx = \frac{1}{a} e^{ax}.}$$

v $\mathbb{R}$

(46) $I_n = \int \sin^n x \, dx$; $n \geq 0$ celé

$I_0 = \int dx = 1$; $I_1 = \int \sin x \, dx = -\cos x$

v $\mathbb{R}$

$$I_2 = \int \sin^2 x \, dx = \int \frac{1}{2}(1 - \cos 2x) dx = \frac{x}{2} - \frac{1}{4} \sin 2x$$

redukční vzorec: (n ≥ 2 celé) $I_n = \int \underbrace{\sin x}_{u'} \cdot \underbrace{\sin^{n-1} x}_{v} \, dx$

$u = -\cos x$; $v' = (n-1) \sin^{n-2} x \cos x$

$$= -\cos x \sin^{n-1} x + (n-1) \int \underbrace{\cos^2 x}_{1 - \sin^2 x} \cdot \sin^{n-2} x \, dx$$

$$I_n = -\cos x \sin^{n-1} x + (n-1) \left[I_{n-2} - I_n \right]$$

$$\boxed{I_n = -\frac{1}{n} \cos x \sin^{n-1} x + \left(\frac{n-1}{n} \right) I_{n-2} \quad \text{v } \mathbb{R}.}$$

(47) $J_n = \int \frac{dx}{\sin^n x} = \int \sin^{-n} x \, dx$; $n \geq 1$ celé

$$J_1 = \int \frac{dx}{\sin x} = \int \frac{dx}{2 \sin\frac{x}{2} \cos\frac{x}{2}} = \int \frac{\cos\frac{x}{2} \, dx}{2 \sin\frac{x}{2} \cos^2\frac{x}{2}} \quad \left| \begin{array}{l} y = \operatorname{tg}\frac{x}{2} \\ dy = \frac{dx}{2\cos^2\frac{x}{2}} \end{array} \right.$$

$$= \int \frac{dy}{y} = \ln|y| = \ln\left|\operatorname{tg}\frac{x}{2}\right| ; \quad x \in (k\pi, (k+1)\pi).$$

$$J_2 = \int \frac{dx}{\sin^2 x} = -\operatorname{cotg} x \; ;$$

rekurentní vzorec (viz příklad (46) pro I_n výše).

$$J_n = I_{-n} \; ; \quad I_{-n} = \frac{-\cos x}{\sin^{n+1}} - (n+1) \cdot \left[I_{-(n+2)} - I_{-n} \right]$$

$$\boxed{J_{n+2} = \frac{-\cos x}{(n+2)\sin^{n+1} x} + \frac{n}{n+1} J_n}$$

(48) $I_n = \int \frac{dx}{(x^2+1)^n}$; $n \geq 1$ celé. $I_1 = \operatorname{arctg} x$ v $\mathbb{R}$

$$I_n = \int \underbrace{1}_{u'} \cdot \underbrace{(x^2+1)^{-n}}_{v} dx = \frac{x}{(x^2+1)^n} + 2n \underbrace{\int \frac{x^2}{(x^2+1)^{n+1}} dx}_{I_n - I_{n+1}}$$

$u = x$; $v' = (-n) \cdot 2x (x^2+1)^{-n-1}$

$$I_n = \frac{x}{(x^2+1)^n} + 2n \left[I_n - I_{n+1} \right]$$

$$\boxed{I_{n+1} = \frac{1}{2n} \cdot \frac{x}{(x^2+1)^n} + \frac{2n-1}{2n} I_n}$$