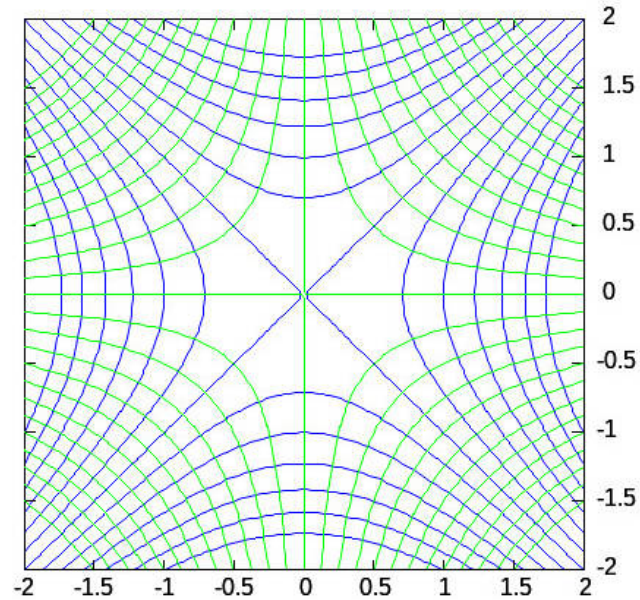
1) $f: z \longmapsto z^2$

$(x,y) \longmapsto (x^2-y^2, 2xy)$



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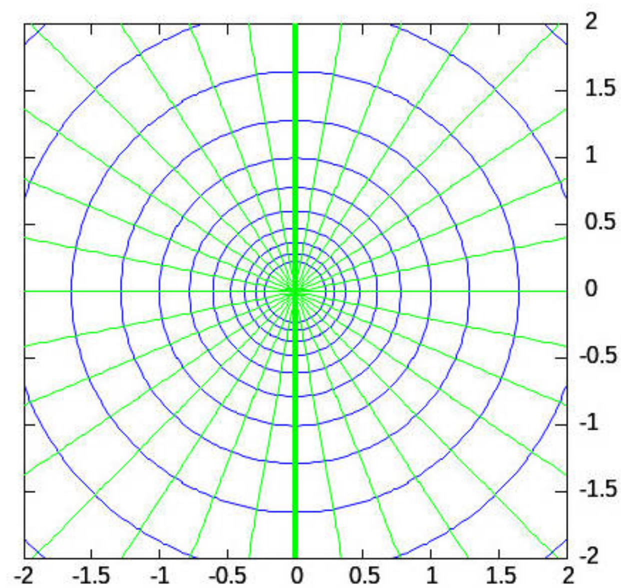
2) $f: z \longmapsto \operatorname{Log} z$

$(x,y) \longmapsto (u,v)$

$u = \ln\sqrt{x^2+y}$

$v = \operatorname{arctg}\left(\frac{y}{x}\right)$

(para $x>0$)



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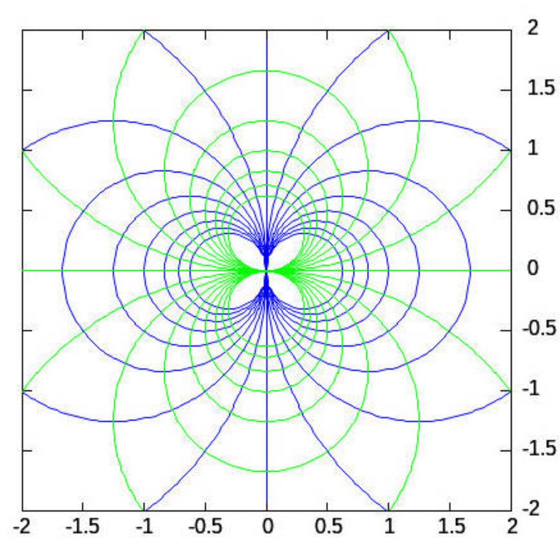
3) $f: z \longmapsto \frac{1}{z}$

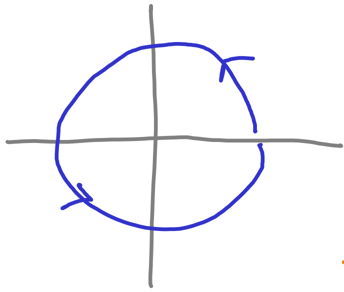
$(x,y) \longmapsto (u,v)$

$u = \dfrac{x}{x^2+y^2}$

$v = \dfrac{-y}{x^2+y^2}$



1

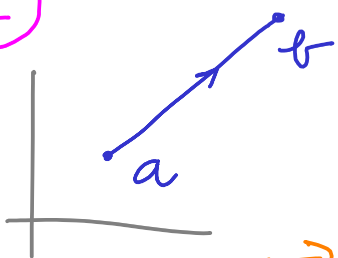


$$\varphi(t)=e^{it},\quad t\in[0,2\pi]$$

$$\varphi'(t)=ie^{it}$$

$$\Rightarrow \int_\varphi \frac{dz}{z}=\int_0^{2\pi}\frac{i\cancel{e^{it}}}{\cancel{e^{it}}}\,dt=2\pi i$$

2



$$\varphi(t)=a+t(b-a),\quad t\in[0,1]$$

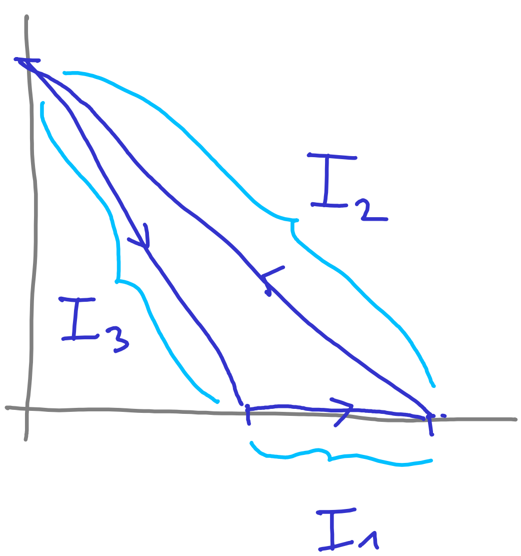
$$\varphi'(t)=b-a$$

$$\Rightarrow \int_\varphi (z^2+1)\,dz=\int_0^1\left(\left(a+t(b-a)\right)^2+1\right)(b-a)\,dt$$

$$=\left(\frac{a^2+ab+b^2}{3}+1\right)(b-a)=\frac{1}{3}\left(b^3-a^3\right)+b-a$$

(viz též d.ú 8, př 5)

4



$$I_1=\int_0^1\frac{dt}{1+t}=\ln 2$$

$$I_2=\int_0^1\frac{(i-2)\,dt}{2+t(i-2)}=-\ln 2+i\underbrace{\left(\operatorname{arctg}2+\operatorname{arctg}\frac{1}{2}\right)}_{\frac{\pi}{2}}$$

$$I_3=-\frac{i\pi}{2}$$