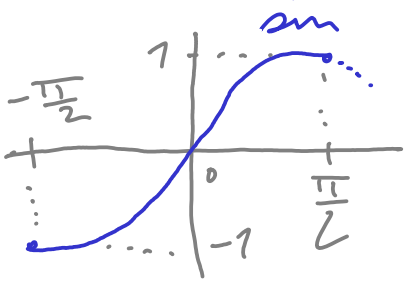
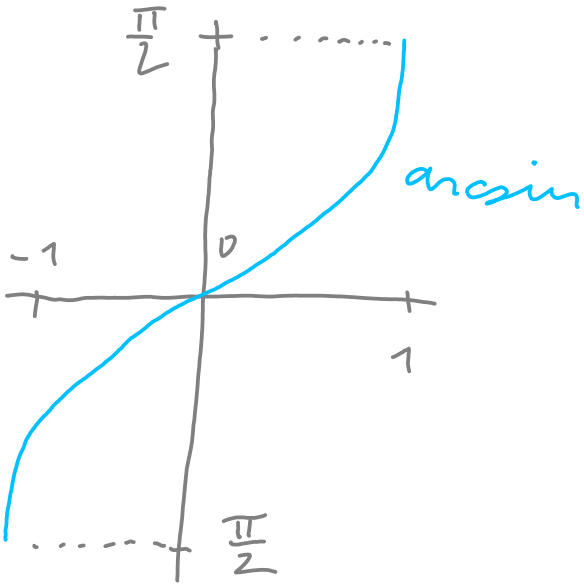
Další elementární fce:

① arcus sinus:

$$\arcsin := \left(\sin|_{[-\frac{\pi}{2},\frac{\pi}{2}]}\right)^{-1}$$





Plyne:

- $\arcsin x$ je rostoucí, prostá na $[-1,1]$, lichá
- $\arcsin 0 = 0$, $\arcsin(\pm 1) = \pm\frac{\pi}{2}$
- základní limita:

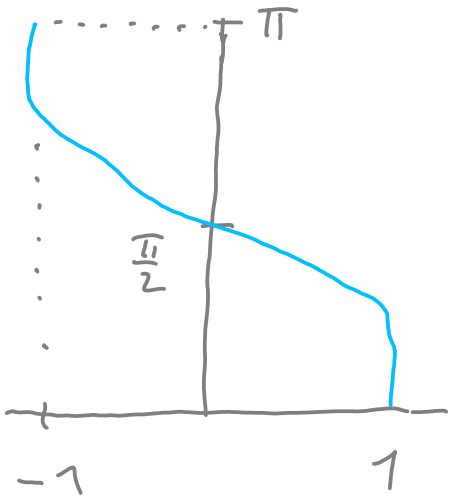
$$\lim_{x\to 0}\frac{\arcsin x}{x} = 1 \quad \Leftarrow \quad \lim_{x\to 0}\frac{\sin x}{x}$$

$$\arcsin'(0) = 1 \quad \Leftarrow \quad \sin'(0) = 1$$

V.4.4

② arcus cosinus:

$$\left(\arccos := \cos|_{[0,\pi]}\right)^{-1}$$



Plocha:

- $\arccos x$ je klesající, rostoucí na $[-1,1]$
- $\arccos(-1)=\pi$, $\arccos(0)=\frac{\pi}{2}$, $\arccos 1=0$
- $\arcsin x+\arccos x=\frac{\pi}{2}$, $x\in[-1,1]$

dk.: $\sin y=\cos\left(\frac{\pi}{2}-y\right)$, $y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\frac{\pi}{2}-y\in[0,\pi]$

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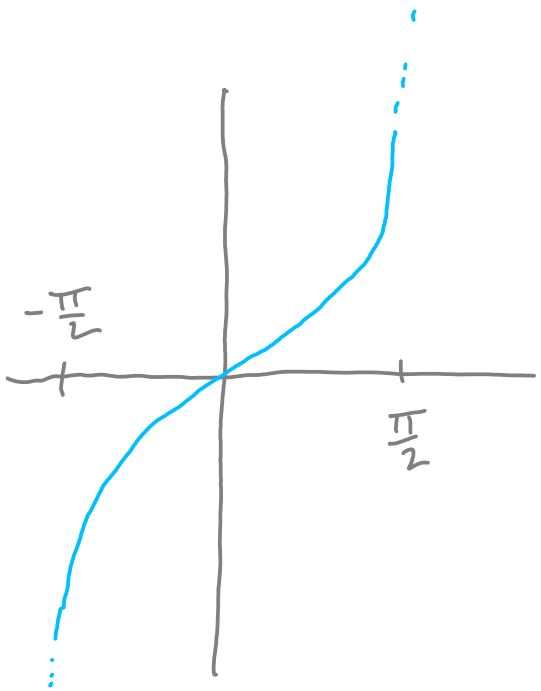
③ tangens:

$$\operatorname{tg} x=\frac{\sin x}{\cos x},\quad x\neq\frac{\pi}{2}+k\pi$$



Plocha:

- liché, π-periodické
- rostoucí v def. oboru

pro $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$: rostoucí, $\mathcal{H} = \mathbb{R}$

$$\lim_{x \to \pm\frac{\pi}{2}\mp} \operatorname{tg} x = \pm\infty$$

dk. Věta 2.8 : $x \to \frac{\pi}{2}-$

$$\Rightarrow \sin x \to \sin\frac{\pi}{2} = 1$$

$$\cos x \to 0, \quad \cos x > 0$$

$$\Rightarrow \frac{1}{\cos x} \to +\infty \quad x \in P_-\left(\frac{\pi}{2}\right)$$

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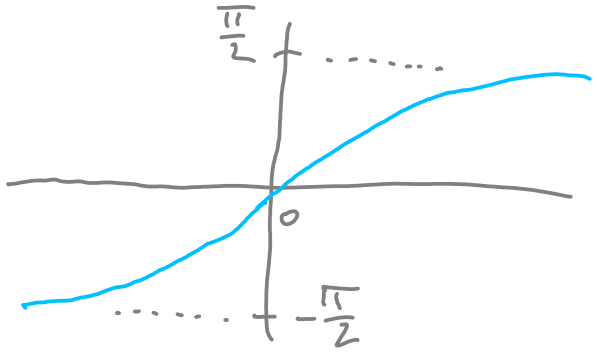
(4) **arcus tangens:**

$$\operatorname{arctg} := \left(\operatorname{tg}|_{\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}\right)_{-1}$$



Plyne:

- $\operatorname{arctg} x$ rostoucí, rostoucí, liché $\nu \mathbb{R}$
- $\operatorname{arctg} 0 = 0$, $\operatorname{arctg} x \longrightarrow \pm\frac{\pi}{2}$, $x \to \pm\infty$

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(5) $\operatorname{cotg} x = \frac{\cos x}{\sin x}$, $x \neq k\pi$

$$\operatorname{arccotg} = \left(\operatorname{cotg}|_{(0,\pi)}\right)_{-1}$$

$$\operatorname{tgh} x = \frac{\sinh x}{\cosh x}$$

$$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right), \quad \cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$$

Def. Funkce se nazve elementární (na určité množině), je-li to:

1. polynom, rac. fce, odmocnina
2. sin, cos, exp, ln
3. arcsin, arccos, arctg, ...
4. cokoliv dalšího, je-li to výsledkem konečného počtu operací $+, -, \cdot, :$ a $\circ$.

Pozn. • $\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\operatorname{argsinh} x = \ln(x + \sqrt{x^2+1})$

• stačí: e^x, $\ln x$, i : $\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$

$\arcsin x = -i \ln(ix + \sqrt{1-x^2})$

• nejsou elementární: Dirichletova fce
Riemannova -"-
Cantorova -"-

$$D(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 0, & x \in \mathbb{R} - \mathbb{Q} \end{cases}$$