(1) $$u(x,t) = \underbrace{\frac{1}{2\pi}\frac{\partial}{\partial t}\int_{|x-y|<t} \frac{u_0(y)\,dy}{\sqrt{t^2-|x-y|^2}}}_{I_1} + \underbrace{\frac{1}{2\pi}\int_{|x-y|<t}\frac{u_1(y)\,dy}{\sqrt{t^2-|x-y|^2}}}_{I_2}$$

$$I_1 = \left|\begin{array}{l} \text{polární souř.} \\ y_1 = x_1 + r\cos u \\ y_2 = x_2 + r\sin u \\ J = r, \quad r\in(0,t) \\ \qquad\quad u\in(0,2\pi) \end{array}\right| = \int_0^t\int_0^{2\pi} \frac{(x_1+r\cos u)^2+(x_2+r\sin u)^2}{\sqrt{t^2-r^2}}\,du\,r\,dr$$

$$= \int_0^t \frac{r\,(|x|^2+r^2)}{\sqrt{t^2-r^2}}\,dr = \left|\begin{array}{l} y=\sqrt{t^2-r^2} \\ r^2 = t^2-y^2 \\ r\,dr = -y\,dy \end{array}\right| = \int_0^t |x|^2+t^2-y^2\,dy$$

$$= |x|^2 t + \frac{2}{3}t^3.$$

$$I_2 = \int_{|x-y|<t}\frac{dy}{\sqrt{t^2-|x-y|^2}} \left|\begin{array}{l} \text{opět} \\ \text{polární} \\ \text{souř.} \end{array}\right| = 2\pi t$$

$$\Rightarrow u(x,t) = \underbrace{\frac{1}{2\pi}\frac{\partial}{\partial t}\left(|x|^2 t + \frac{2}{3}t^3\right)}_{|x|^2+2t^2} + \underbrace{\frac{1}{2\pi}(2\pi t)}_{t}$$

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(2) $$u(x,t) = \underbrace{\frac{\partial}{\partial t}\left(\frac{1}{4\pi t}\int_{|x-y|=t} u_0(y)\,d\sigma(y)\right)}_{I_1} + \underbrace{\frac{1}{4\pi t}\int_{|x-y|=t} u_1(y)\,d\sigma(y)}_{I_2}$$

$$I_1 = \int_{|x-y|=t} |y|^2\,d\sigma \left|\begin{array}{l} \text{sférické souř.} \\ y_1 = x_1 + t\cos u\cos v \\ y_2 = x_2 + t\cos u\sin v \\ y_3 = x_3 + t\sin u \end{array}\right. \qquad \begin{array}{l} u\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \\ v\in(0,2\pi) \\ d\sigma = t^2\cos u\,du\,dv \end{array}$$

$'' \in \mathbb{R}^3$

$$|y|^2 = |x+tR|^2 = |x|^2 + t\langle x,R\rangle + t^2\underbrace{|R|^2}_{1}$$

$$I_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\pi} \Big(|x|^2 + t\underbrace{\langle x,R\rangle}_{=0} + t^2\Big) t^2 \cos u \, dv \, du$$

integrace dle dv lichost

$$= 2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(|x|^2 t^2 + t^4\right) \cos u \, du = 4\pi\left(|x|^2 t^2 + t^4\right)$$

$$I_2 = \int_{|x-y|=t} y_3 \, d\sigma = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\pi} \Big(x_3 + \underbrace{t \sin u\Big) t^2 \cos u}_{=0} \, dv \, du$$

integrace dle du lichost

$$= 2x_3 t^2 \cdot 2\pi = 4\pi x_3 t^2$$

$$\Rightarrow \quad u(x,t) = \underbrace{\frac{\partial}{\partial t}\left(\frac{1}{4\pi t} \cdot 4\pi\left(|x|^2 t^2 + t^4\right)\right)}_{|x|^2 + 3t^2} + \underbrace{\frac{1}{4\pi t}\left(4\pi x_3 t^2\right)}_{x_3 t}$$