① [**Adam Zaplatílek**]
Let $B$ be a closed ball in $R^n$. Show that the dynamical system $(\varphi, B)$ has a stationary point. Deduce that any periodic orbit in $R^2$ has at least one stationary point in its interior.

② [**Michael Zelina**]
Show that the problem

$$\begin{aligned} x' &= \sin y \\ y' &= x + u \end{aligned}$$

is globally controllable.

③ [**Mikuláš Zindulka**]
Consider the control problem

$$\begin{aligned} x' &= \cos 2\pi u \\ y' &= \sin 2\pi u \\ z' &= -1 \end{aligned}$$

Show that $(x_0, y_0, z_0) = (0, 0, 1) \in \mathcal{R}(1)$; in fact, there are many controls that bring this initial state to zero at $t = 1$.
However, show that the cost functional

$$P[u] = \int_0^1 x^2(t) + y^2(t)\, dt$$

does not attain its minimum.
What is the geometric meaning of the problem?

Hints.

1. Apply Brouwer's Theorem to $F_n(x) = \varphi(1/2^n, x)$ to find an orbit of period $1/2^n$. By taking a subsequence, find an orbit with arbitrarily small period. Show that this must be a stationary point.

2. Use linearization to show local controllability. By elementary qualitative analysis investigate solutions for various values of $u$ to obtain the global case.

   Cf. also úloha 4, page 5 of Kapitola 14 sbírky pcODR.

3. Observe that by the $(x, y)$ components stay close to $(0, 0)$ for $u(t) = kt$, $k > 0$ large, and for suitable $k > 0$ attains the final condition exactly.

   Show further that the infimum of $P[\cdot]$ is zero, which however cannot be attained.