ODE2 – 2nd term – 26 Jan 2017

Problem 1 [7 pts] Consider the system

$$X' = \epsilon X + F(X)$$

where $X = (x, y)^T$ and $F = (F_1, F_2)^T$ with

$$F_1 = \frac{-x^5}{x^2 + 2y^2 + 1}$$
$$F_2 = \frac{-y^5}{2x^2 + y^2 + 1}$$

(a) Show that for $\epsilon \leq 0$ there is no (nontrivial) periodic solution.
(b) Show that for any $\epsilon > 0$, there is at least one (nontrivial) periodic solution
Hint: (a) Bendixson-Dulac. (b) Show that sufficiently large ball is positively invariant; apply Poincaré-Bendixson

Problem 2 [5 pts] Consider the system

$$X' = AX + BU$$

where $X = (x, y)^T$, $U = (u, v)^T$ and A, B are antisymmetric and symmetric, respectively, i.e.

$$A = \begin{pmatrix} 0 & -a \\ a & 0 \end{pmatrix} \qquad B = \begin{pmatrix} b & c \\ c & b \end{pmatrix}$$

for some real parameters a, b, c.
(a) Determine conditions under which the system is globally controllable.
(b) Determince conditions, under which the system $X' = AX$ is NOT observable via $P = x + y$.

Problem 3 [8 pts] Show that the system (with a real parameter a)

$$x' = ax^2 - y^3$$
$$y' = -y + x^2$$

has a centre manifold of the form $y = \phi(x)$ in some neighborhood of $(x, y) = (0, 0)$. Find a suitable approximation of $\phi(x)$ to determine the stability of the origin.