d.ú. 2/13

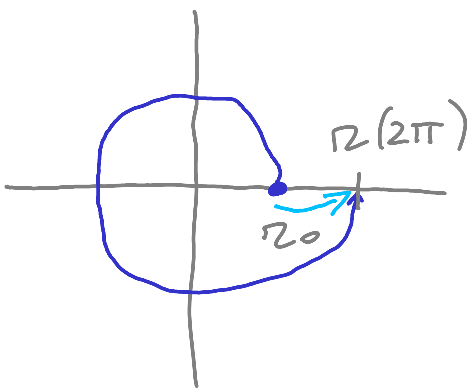
12.11.2020

$$x' = ax - y + xy^2$$
$$y' = x + ay + y^3 \qquad a \in \mathbb{R}$$

? $\exists$ per. řešení (elementární – pol. souř.)

$$r' = ar + \underline{r^3 \sin^2 \phi}$$
$$\phi' = 1 \Rightarrow \phi = t + \cancel{c}$$

BÚNO



per. orbit $\Leftrightarrow r(2\pi) = r(0)$

$\Leftrightarrow$ pevný bod Poincarého zobrazení

i) $a \geq 0$: $r' > 0$ o.r., tj. roste,

$\Rightarrow \nexists$ per. orbit

ii) $a < 0$: Bernoulliho rce : $\rho := r^{-2}$

$$\frac{r'}{r^3} = \frac{a}{r^2} + \sin^2 t$$

$$-\frac{1}{2}\rho' = a\rho + \sin^2 t$$

$$\rho' + 2a\rho = -2\sin^2 t \quad \Big/ \text{if} \cdot e^{+2a}$$

$$\underbrace{\rho(2\pi)}_{\overset{?}{=}\,\rho(0)} \cdot e^{4a\pi} - \rho(0) = -2\underbrace{\int_0^{2\pi} e^{2a\tau} \sin^2\tau \, d\tau}$$

$$\left(e^{4a\pi} - 1\right)\rho(0) = -2\, \overset{\shortparallel}{I}$$

$$\rho(0) = \frac{-2I}{e^{4a\pi}-1} > 0$$

$\Downarrow$

$\exists!$ per. řešení

$$r(0) = \frac{1}{\sqrt{\rho(0)}}$$

(nez. l nez.)

(Eloquence lif.)

S.L.3 řešte

$$\boxed{\begin{aligned} x'' + \cancel{\lambda} x &= h(t), \quad t \in [0,T] \\ x(0) = x(T) &= 0 \end{aligned}} \quad (*) \qquad \lambda = 0$$

$0 \notin \sigma(\mathcal{L})$:

$$\left[\begin{aligned} u(t) &= t \\ v(t) &= t - T \end{aligned}\right] \quad \text{-- řeší } x'' = 0$$

$\circ \; x(0) = 0$

resp. $x(T) = 0$

$$c = p(uv' - u'v) = t - (t-T) = T$$

$$G_0(t,s) = \begin{cases} \frac{1}{T}(t-T)s, & s < t \\ \frac{1}{T}(s-T)t, & s > t \end{cases}$$

$$w(t) = \int_0^T G_0(t,s)\, h(s)\, ds =$$

$$= \frac{t-T}{T} \int_0^t s\, h(s)\, ds + \frac{t}{T} \int_t^T (s-T)\, h(s)\, ds$$

d. d. cv.: ověřte, že $w(t)$ řeší (*)