Příkl. 5

$$\varepsilon' = 0$$

$$x' = \varepsilon x - x^3 + xy \quad \Big\}\ \text{centr.}$$

$$y' = -y + y^2 - x^2 \quad \Big\}\ \text{stab.}$$

ε malé

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linearizace: $M = \begin{pmatrix} \varepsilon & 0 \\ 0 & -1 \end{pmatrix}$

TRIK: falešné rovnice: $A = \begin{pmatrix} 0, & 0 \\ 0, & 0 \end{pmatrix}$; $B = (-1)$

$n = 2$ $\qquad m = 1$

$$f = (0,\ \varepsilon x - x^3 + xy)$$

$$g = y^2 - x^2$$

víme: $\exists$ c.v. $y = \phi(\varepsilon, x)$

aproximace?

$$y' = \frac{\partial \phi}{\partial \varepsilon} \cdot \varepsilon' + \frac{\partial \phi}{\partial x} \cdot x'$$

$$-\phi + \phi^2 - x^2 = \frac{\partial \phi}{\partial x}\left(\varepsilon x - x^3 + x\phi\right)$$

$$\Rightarrow \quad M\phi = \frac{\partial \phi}{\partial x}\left(\varepsilon x - x^3 + x\phi\right) + \phi + x^2 + \phi^2$$

?

i) zkusme: $\phi = 0$, tj. $M0 = x^2 = O(x^2)$

tj: $\phi(x, \varepsilon) = 0 + O(|X|^2)$, kde $X = (x, \varepsilon)$

Red. rce: $(\varepsilon' = 0)$

$$p' = \varepsilon p - p^3 + p\,\phi(p,\varepsilon)$$

$$= \varepsilon p - p^3 + p\,(\mathcal{O}(|p|^2))$$

$$\Rightarrow \quad p' = p\,(\varepsilon - p^2 + \mathcal{O}(|p|^2))$$

?! ... nestačí!

ii) Zkusme: $\psi(x,\varepsilon) = -x^2$

$$\Rightarrow \quad \Pi\psi = -2x\,(\varepsilon x - x^3 + x\cdot(-x^2)) + (-x^2)^2$$

$$= \mathcal{O}(|X|^3), \text{ kde } X = (x,\varepsilon)$$

tj. členy $x^3, x^2\varepsilon, x\varepsilon^2, \varepsilon^3$

Red. rce: $p' = \varepsilon p - p^3 + p\,(-p^2 + \mathcal{O}(|p|^3))$

$$p' = p\,(\varepsilon - 2p^2 + \mathcal{O}(|p|^3))$$

vedoucí člen

viz Ex. 19

$$\boxed{p' = p\,(\varepsilon - 2p^2)}$$

bifurkační diagram

stac. body stab.

stac. body nestab.

řešení (ε dané)

• bod bifurkace