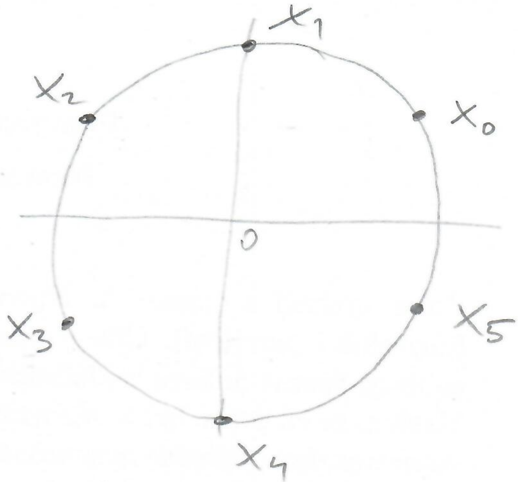
Př.: $\int \frac{x^4+1}{x^6+1}\,dx$



(i) De Moivre: $x^6 = -1 = e^{i\pi + 2k\pi i}$

$x = x_k = \exp\left(\frac{i\pi}{6} + \frac{ik\pi}{3}\right)$,

$x_0 = \exp\left(\frac{i\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}\sqrt{3} + \frac{i}{2}$

$x_1 = \exp\left(\frac{i\pi}{2}\right) = i$

$x_2 = \exp\left(5\frac{i\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}\sqrt{3} + \frac{i}{2}$

$x_3 = \overline{x_2} = -\frac{1}{2}\sqrt{3} - \frac{i}{2}$, $x_4 = \overline{x_1} = -i$, $x_5 = \overline{x_0} = \frac{1}{2}\sqrt{3} - \frac{i}{2}$

$$x^6+1 = (x-x_0)(x-\overline{x_0})\cdot(x-x_1)(x-\overline{x_1})\cdot(x-x_2)(x-\overline{x_2})$$
$$= (x^2-\sqrt{3}x+1)\cdot(x^2+1)\cdot(x+\sqrt{3}x+1).$$

(ii) TRIK: $x^6+1 = y^3+1 = (y+1)(y^2-y+1) \quad / \; y = x^2$
$$= (x^2+1)\cdot(x^4-x^2+1).$$

$$x^4-x^2+1 = (x^2+1)^2 - 3x^2 = (x^2+1)^2 - (\sqrt{3}x)^2 = A^2 - B^2$$
$$= (A+B)(A-B) = (x^2+1+\sqrt{3}x)(x^2+1-\sqrt{3}x).$$

(iii) rozklad: $\frac{x^4+1}{x^6+1} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2-\sqrt{3}x+1} + \frac{Ex+F}{x^2+\sqrt{3}x+1}$

$\Rightarrow A = C = E = 0$
$B = \frac{2}{3}$, $D = F = \frac{1}{6}$; $\int \frac{dx}{x^2 \pm \sqrt{3}x+1} = \int \frac{4}{(2x\pm\sqrt{3})^2+1} = 2\,\mathrm{arctg}(2x\pm\sqrt{3})$

CELKEM: $\frac{2}{3}\,\mathrm{arctg}\,x + \frac{1}{2}\,\mathrm{arctg}(2x-\sqrt{3}) + \frac{1}{2}\,\mathrm{arctg}(2x+\sqrt{3})$, $x \in \mathbb{R}$