Př. Sečtěte řadu $\sum\limits_{k=1}^{\infty}(-1)^k \frac{k x^k}{(2k-1)!}$

$$S(x) = x \sum_{k=1}^{\infty}(-1)^k \frac{k x^{k-1}}{(2k-1)!} = x \underbrace{\left(\sum_{k=1}^{\infty}(-1)^k \frac{x^k}{(2k-1)!}\right)'}_{T(x)}.$$

i) $x>0$: $\sqrt{x} = y$, $y^2 = x$

$$T(x) = T(y^2) = \sum_{k=1}^{\infty}(-1)^k \frac{y^{2k}}{(2k-1)!} = y \underbrace{\sum_{k=1}^{\infty}(-1)^k \frac{y^{2k-1}}{(2k-1)!}}_{\overset{\shortparallel}{-\sin y}}$$

$$= -y \sin y,$$

CELKEM: $S(x) = x\left(-\sqrt{x}\cdot \sin\sqrt{x}\right)'$, $x>0$

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ii) $x<0$: $\sqrt{-x} = y$, $y^2 = -x$

$$T(x) = T(-y^2) = \sum_{k=1}^{\infty}(-1)^k \frac{(-y^2)^k}{(2k-1)!} = y \underbrace{\sum_{k=1}^{\infty} \frac{y^{2k-1}}{(2k-1)!}}_{\sinh y}$$

$$= y \sinh y$$

CELKEM: $S(x) = x\left(\sqrt{-x}\, \sinh\sqrt{-x}\right)'$, $x<0$.

Př.: Vypočtěte konv. mocn. řady $\sum\limits_{k=1}^{\infty}(-1)^k \underbrace{\frac{k^k}{k!}}_{a_k} x^k$

i) poloměr konv.:

$$\left|\frac{a_{k+1}}{a_k}\right| = \frac{(k+1)^{k+1}}{(k+1)!}\cdot\frac{k!}{k^k} = \left(\frac{k+1}{k}\right)^k \to e$$

$$\Rightarrow \rho = \frac{1}{e}$$

ii) $x = -\frac{1}{e}$: $\Rightarrow \sum b_k$, $\quad b_k = \left(\frac{k}{e}\right)^k \frac{1}{k!}$

Pozn: Stirlingův vzorec: $k! \sim \left(\frac{k}{e}\right)^k \sqrt{2\pi k}$

$\Rightarrow b_k \sim \frac{1}{\sqrt{k}}$, tj. $\sum_k b_k$ div.

Raabe: $k\left(\frac{b_k}{b_{k+1}} - 1\right) = k\left(\frac{e}{(1+\frac{1}{k})^k} - 1\right) = f\left(\frac{1}{k}\right)$

kde $f(x) = \frac{1}{x}\left(\exp\left(1 - \frac{1}{x}\ln(1+x)\right) - 1\right)$.

Taylor: $\ln(1+x) = x - \frac{x^2}{2} + o(x^2)$, $x \to 0$

$1 - \frac{1}{x}\ln(1+x) = \frac{x}{2} + o(x)$

$\exp\left(\frac{x}{2} + o(x)\right) = 1 + \frac{x}{2} + o(x)$

$\Rightarrow f(x) = \frac{1}{x}\left(\frac{x}{2} + o(x)\right) = \frac{1}{2} + o(1) \to \underline{\underline{1/2}}$

Raabe $\Rightarrow \sum b_k$ div, neboť $1/2 < 1$

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iii) $x = \frac{1}{e} \Rightarrow \sum (-1)^k b_k$

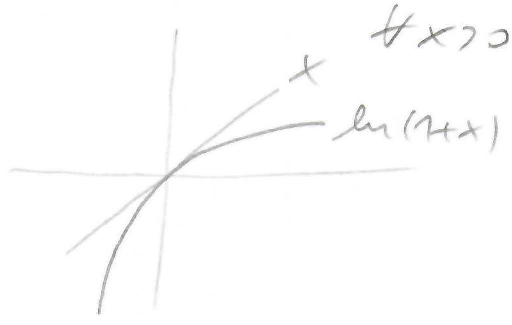
? Leibniz: $b_k \to 0$, klesá.

$\frac{b_{k+1}}{b_k} \overset{?}{<} 1$

$\left(\frac{k+1}{k}\right)^k < e \quad / \ln$

$\ln\left(1+\frac{1}{k}\right) < \frac{1}{k}$

OK, neboť $\ln(1+x) < x$ $\quad \forall x > 0$



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složité: $b_k \to 0 \Leftrightarrow \ln b_k \to -\infty$

$\ln b_k = k \ln\frac{k}{e} - \ln k!$

$= k(\ln k - 1) - \sum_{j=1}^{k} \ln j$

potřebuji <u>dolní</u> odhad

dokončení :

1\. pokus: $\ln n! = \sum_{j=1}^{n} \ln j \to \int_1^n \ln x \, dx$

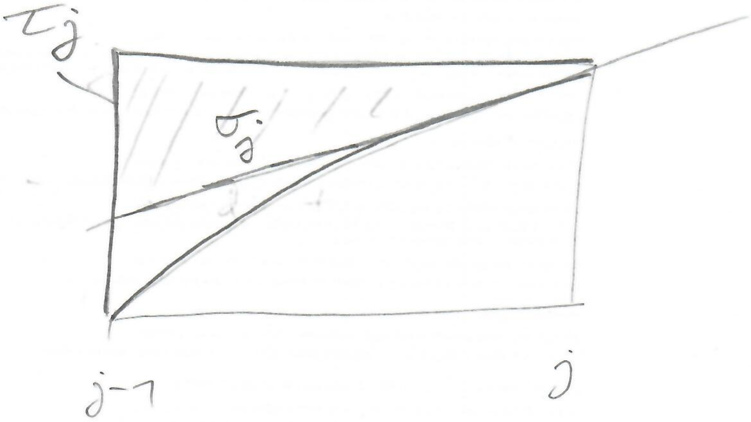
$$= \left[x \ln x \right]_1^n - \int_1^n 1 \, dx$$

$$= n \cdot \ln n - (n-1)$$

ale to nestačí: $\ln b_n < n \ln(n-1) - \left(n \cdot \ln n - (n-1) \right) = -1$

2\. pokus – přesnější odhad:

$$\sum_{j=1}^{n} \ln j = \int_1^n \ln x \, dx + \sum_{j=1}^{n} \tau_j$$



$$\tau_j > \sigma_j = \frac{1}{2j}$$

$$\Rightarrow \sum_{j=1}^{n} \ln j > n \cdot \ln n - (n-1) + \sum_{j=1}^{n} \frac{1}{2j}$$

$$\Rightarrow \ln b_n < -1 - \sum_{j=1}^{n} \frac{1}{2j} \to -\infty, \quad n \to \infty.$$