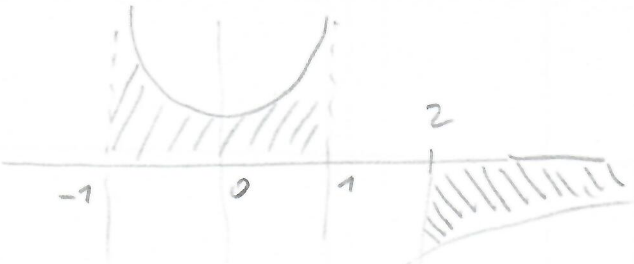
1) $\int_{-1}^{1} \frac{dx}{1-x^2}$ resp. $\int_{2}^{+\infty} \frac{dx}{1-x^2}$



prim. fce: $\int \frac{dx}{1-x^2} = \frac{1}{2} \int \frac{1}{1-x} + \frac{1}{1+x}\, dx = \frac{1}{2} \ln\left|\frac{1+x}{1-x}\right|$,

$x \in (-1,1), (1,+\infty)$
$(-\infty,-1)$.

(i) $+\infty - (-\infty) = +\infty$

(ii) $0 - \frac{1}{2} \ln 3$

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2) $I = \int_{0}^{+\infty} \underbrace{e^{-ax}}_{u'} \underbrace{\cos bx}_{v}\, dx$ per-partes $= \underbrace{\left[-\frac{1}{a} e^{-ax} \cdot \cos bx\right]_{0}^{+\infty}}_{=} - \frac{b}{a} \int_{0}^{+\infty} e^{-ax} \sin bx$

$u = \frac{-e^{-ax}}{a}$,

$v' = -b \sin bx$

$0 - \left(-\frac{1}{a}\right) = \frac{1}{a}$, neboť $e^{-ax} \to 0$, $\cos bx$ omezené pro $x \to +\infty$.

$\Rightarrow$ $\boxed{I = \frac{1}{a} - \frac{b}{a} J \quad J = \frac{b}{a} I}$ $\Rightarrow$ $\boxed{I = \frac{a}{a^2+b^2},\ J = \frac{b}{a^2+b^2}}$

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3) $\int_{0}^{\frac{\pi}{2}} \frac{2 \sin t \cos t}{\sqrt{\sin^2 t + 3 \sin t + 1}}$ | $\sin t = u \in (0,1)$, $\cos t\, dt = du$ | $\boxed{\doteq 0{,}5554\ldots}$

$= \int_{0}^{1} \underbrace{\frac{2u\, du}{\sqrt{u^2+3u+1}}}_{f(u)}$;

rozklad jmenovatele:

$u^2+3u+1 = (u-u_1)(u-u_2)$,

$u_{1,2} = \frac{1}{2}\left(-3 \pm \sqrt{5}\right) < 0$

Pozorují: $\left(\sqrt{u^2+3u+1}\right)' = \dfrac{2u+3}{2\sqrt{u^2+3u+1}} = \dfrac{u+3/2}{\sqrt{u^2+3u+1}}$,

$$F(u) = \int \frac{2u\,du}{\sqrt{u^2+3u+1}} = 2\int \frac{u+3/2}{\sqrt{u^2+3u+1}}\,du - \frac{3}{2}\underbrace{\int \frac{du}{\sqrt{u^2+3u+1}}}_{\tilde{F}(u)}$$

$$\tilde{F}(u) = \int \frac{du}{\sqrt{(u-u_1)(u-u_2)}} = \int \frac{du}{(u-u_1)\sqrt{\frac{u-u_2}{u-u_1}}}$$, pro (BÚNO) $u>0>u_{1,2}$

substituce: $u_{1,2} = \frac{1}{2}\left(-3\pm\sqrt{5}\right)$

$$t = \sqrt{\frac{u-u_2}{u-u_1}} \iff u = \frac{t^2 u_1 - u_2}{t^2-1}$$

$$u-u_1 = \frac{u_1-u_2}{t^2-1}, \quad du = \frac{2t}{(t^2-1)^2}(u_2-u_1)$$

$$\rightsquigarrow \int \frac{t^2-1}{u_1-u_2}\cdot\frac{1}{t}\cdot\frac{2t}{(t^2-1)^2}(u_2-u_1)\,dt$$

$$= \int \frac{2}{1-t^2}\,dt = \int \frac{1}{1+t}+\frac{1}{1-t}\,dt$$

$$= \ln\left|\frac{1+t}{1-t}\right|$$

$$\Rightarrow \tilde{F}(u) = \ln\left|\frac{1+\sqrt{\frac{u-u_2}{u-u_1}}}{1-\sqrt{\frac{u-u_2}{u-u_1}}}\right|$$

úprava:
$$\frac{1+\sqrt{\frac{u-u_2}{u-u_1}}}{1-\sqrt{\frac{u-u_2}{u-u_1}}} = \frac{\sqrt{u-u_1}+\sqrt{u-u_2}}{\sqrt{u-u_1}-\sqrt{u-u_2}} \cdot \frac{\sqrt{}+\sqrt{}}{\sqrt{}+\sqrt{}}$$
$$= \frac{1}{u_2-u_1}\cdot\left(\sqrt{u-u_1}+\sqrt{u-u_2}\right)^2 = \frac{2}{u_2-u_1}\left(u-\frac{u_1+u_2}{2}+\sqrt{(u-u_1)(u-u_2)}\right)$$

v našem případě: $\tilde{F}(u) = \ln\frac{2}{\sqrt5}\left(u-\frac{3}{2}+\sqrt{u^2+3u+1}\right)$.

CELKEM: $F(u) = 2\sqrt{u^2+3u+1} - \frac{3}{2}\ln\left(2u-3+2\sqrt{u^2+3u+1}\right)$

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4) $\displaystyle\int_0^1 \frac{dx}{1+\sqrt{x^2+x+1}} =$ | Euler: $\sqrt{x^2+x+1} = t-x$

$t = x+\sqrt{x^2+x+1} \in (1, 1+\sqrt3)$

$x = \frac{2t+1}{t^2-1}$, $dx = \frac{2(t^2+t+1)}{(2t+1)^2}$

$$= \int_1^{1+\sqrt3} g(t)\,dt;\quad \text{kde } g(t) = \frac{2(t^2+t+1)}{(2t+1)(t^2+3t+2)}$$

$$\boxed{I \doteq 0{,}4315\ldots}$$

$$= \frac{2}{2t+1}+\frac{2}{t+2}-\frac{2}{t+1}$$

$$G(t) = \int g(t) = \ln(2t+1)+2\ln\left(\frac{t+2}{t+1}\right),\ t>0$$

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5) $\displaystyle\int_0^\pi \sin^4 x\,dx = \frac{3}{8}\pi$; pomocí vzorců $\sin^2 y = \frac12(1-\cos 2y)$,

$\cos^2 y = \frac12(1+\cos 2y)$

$$\Rightarrow \sin^4 x = \frac18\left(\cos 4x - 4\cos 2x + 3\right)$$