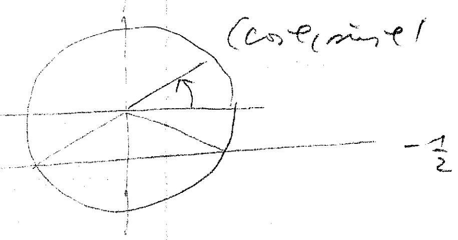
1) $\frac{5+\sin x}{1-\sin x} = 3$ $\qquad \sin x \neq 1$

$x = \frac{\pi}{2} + 2k\pi$

$5 + \sin x = 3 - 3\sin x$

$4 \sin x = -2$

$\sin x = -\frac{1}{2}$; $\boxed{\begin{aligned} x &= -\frac{\pi}{6} + 2k\pi \\ &\ \ \frac{7\pi}{6} + 2k\pi \end{aligned}}$



2) $2\,\mathrm{tg}\, x - 3\,\mathrm{cotg}\, x = 1$ $\qquad \cos x \neq 0 \ : \ x \neq \frac{\pi}{2} + k\pi$

$2\frac{\sin x}{\cos x} - 3\frac{\cos x}{\sin x} = 1$ $\qquad \sin x \neq 0 \ : \ x \neq k\pi$

$\qquad$ tj. : $x \neq \frac{k\pi}{2}$; $k \in \mathbb{Z}$

$2\sin^2 x - 3\cos^2 x = \sin x \cos x$

$2t - 3/t = 1$ $\qquad ; t = \mathrm{tg}\, x$

$2t^2 - t - 3 = 0$; $D = 1 + 4\cdot 3\cdot 2 = 25$

$t_{1,2} = \frac{1 \pm 5}{4} = \begin{cases} 3/2 \\ -1 \end{cases}$

$\mathrm{tg}\, x = 3/2$:

$\mathrm{tg}\, x = -1$: $x = -\frac{\pi}{4} + k\pi$

aside: $\mathrm{tg}\, x = \frac{\sin x}{\sqrt{1-\sin^2 x}} = \frac{3}{2}$

$\frac{\sin^2 x}{1-\sin^2 x} = \frac{9}{4}$

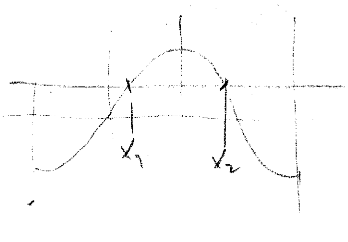
$\sin^2 x = \frac{9}{4} - \frac{9}{4}\sin^2 x$

$\frac{13}{4}\sin^2 x = \frac{9}{4}$

$\sin x = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}}$ $\qquad \boxed{\begin{aligned} x &= \arcsin\frac{3}{\sqrt{13}} + 2k\pi \\ x &= \pi - \arcsin\frac{3}{\sqrt{13}} + 2k\pi \end{aligned}}$

Pozn. $\sin x = a$: i) $|a| > 1$; tj. $a \in (-\infty, -1) \cup (1, +\infty)$

$\emptyset$



ii) $a \in [-1, 1]$: $x_1 = \arcsin a$; $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin(x_2 - \pi) = -a$

$x_2 - \pi = \arcsin(-a)$

3) $\sin^2 x - \cos^2 x + \sin x = 0$

$s^2 - (1 - s^2) + s = 0$

$2s^2 + s - 1 = 0$

$D = 1 + 8 = 9$

$s_{1,2} = \frac{-1 \pm 3}{4} = \left\langle \begin{array}{l} \frac{1}{2} \\ -1 \end{array} \right.$

$$\boxed{\begin{array}{l} x = \frac{\pi}{4} + 2k\pi \\ \quad \frac{3\pi}{4} + 2k\pi \\ x = -\frac{\pi}{2} + 2k\pi \end{array}}$$

4) $\frac{\sin^2 x}{\operatorname{tg} x} + \cos^2 x \cdot \operatorname{tg} x = \frac{1}{2}$; $x \neq \frac{\pi}{2} + k\pi$

$\neq \pi + k\pi$

$\sin x \cdot \cos x + \cos x \cdot \sin x = \frac{1}{2}$

$2 \sin x \cos x = \frac{1}{2}$

$\sin 2x = \frac{1}{2}$;

$2x = \frac{\pi}{6} + 2k\pi$

$= \frac{5\pi}{6} + 2k\pi$

$$\boxed{\begin{array}{l} x = \frac{\pi}{12} + k\pi \\ \quad \frac{5\pi}{12} + k\pi \end{array}}$$