MAI 1

1. Dokazte:

   (a) $1 + 2 + \dots + n = \frac{1}{2}n(n+1)$
   (b) $1^2 + 2^2 + \dots n^2 = \frac{1}{6}n(n+1)(2n+1)$
   (c) $1^3 + 2^3 + \dots n^3 = (1 + 2 + \dots n)^2$
   (d) $1 + q + q^2 + \dots q^n = \frac{1-q^{n+1}}{1-q}, \qquad q \neq 1$
   (e) $\sum_{k=2}^n \frac{1}{k^2} < \frac{n-1}{n+1}$
   (f) $\sum_{k=1}^n \frac{1}{k} \leq 2\sqrt{n}$
   (g) $(1+x)^n \geq 1 + nx, \quad x \geq -1$
   (h) $\sum_{k=1}^n \frac{1}{\sqrt{k}} > \sqrt{n}, \quad n \geq 2$

2. Reste nerovnice:

   (a) $(x-3)(x+1)(3x+6) > 0$
   (b) $\frac{2x+1}{x-3} > 4$
   (c) $|x+1| + |x-2| \leq 4$
   (d) $\frac{x-1}{x-2} > \frac{x-3}{x-4}$
   (e) $\frac{1}{x} + \frac{1}{1-x} < 2$
   (f) $|x^2 - 4x + 3| \leq |x^2 - 4|$
   (g) $\frac{|x+1|}{x-1} \geq x$

3. Dokazte rovnosti:

   (a) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
   (b) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
   (c) $C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B)$
   (d) $C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$
   (e) $A \setminus (A \setminus B) = A \cap B$
   (f) $A \setminus (B \cup C) = (A \setminus B) \setminus C$

4. Zkoumejte vztah nasledujicich mnozin:

   (a) $f(A \cup B)$ a $f(A) \cup f(B)$
   (b) $f(A \cap B)$ a $f(A) \cap f(B)$
   (c) $f(f^{-1}(A))$ a A
   (d) $f^{-1}(f(A))$ a A
   (e) $f^{-1}(A \cup B)$ a $f^{-1}(A) \cup f^{-1}(B)$
   (f) $f^{-1}(A \cap B)$ a $f^{-1}(A) \cap f^{-1}(B)$

5. Urcete maximum, minimum, infimum a supremum mnozin:

   (a) $M := \{1/(x-1) : \ x \geq 0\}$
   (b) $M := \{\frac{n}{n+1} : \ n \in N\}$
   (c) $M := \{\sin \frac{n\pi}{4} : \ n \in N\}$
   (d) $M := \left\{(-1)^n \cdot \frac{n^2+1}{n^2-1} : \ n \in N\right\}$
   (e) Ukazte, ze $\sup\{-a : \ a \in A\} = -\inf A$.
   (f) Ukazte, ze $\sup\{a+b : \ a \in A, \ b \in B\} = \sup A + \sup B$.

6. Dokazte nasledujici vlastnosti absolutni hodnoty:

   (a) $|ab| = |a||b|$
   (b) $|a/b| = |a|/|b| \quad (b \neq 0)$
   (c) $-|a| \leq a \leq |a|$
   (d) $|a| = a \operatorname{sgn} a$
   (e) $|a \pm b| \leq |a| + |b|$
   (f) $|a \pm b| \geq ||a| - |b||$

DULEZITE LIMITY

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x \to 0} \frac{\exp x - 1}{x} = 1 \qquad \lim_{x \to \infty} \frac{\ln x}{x} = 0$$

$$\lim_{x \to \infty} \frac{x}{\exp x} = 0$$