A. Vyšetřete konvergenci integrálů:

$$1. \int_0^\infty \frac{\sqrt[3]{x}+3}{\sqrt{x}+2}\,dx \qquad 2. \int_0^1 \frac{\sin(\pi x)}{x^{9/8}\ln x}\,dx \qquad 3. \int_0^\infty \frac{\ln(x+1)}{x^{101/100}}\,dx$$

$$4. \int_0^\infty \frac{\sin^2 x}{x^a}\,dx \qquad 5. \int_0^\infty \sin^3\left(\frac{1}{\sqrt{x}}\right)dx \qquad 6. \int_0^\pi \frac{dx}{\sqrt{\sin x}}$$

$$7. \int_0^\infty x\exp(-x)\sin x\,dx \qquad *8. \int_0^\infty \frac{x}{1+x^5\sin^2 x}\,dx \qquad 9. \int_0^{1/2} \frac{dx}{x\ln^b x}$$

$$10. \int_0^\infty \frac{dx}{\exp x - \cos x} \qquad 11. \int_0^\infty \frac{\operatorname{arctg} x}{x^a}\,dx \qquad *12. \int_0^\infty \frac{p(x)}{q(x)}\,dx$$

B. Záměnou limity a integrálu vypočtěte:

$$1. \lim_{n\to\infty} \int_0^\infty \exp(-x^n)\,dx \qquad 2. \lim_{n\to\infty} \int_0^{10} \frac{nx}{1+n^2x^2}\,dx$$

$$3. \lim_{n\to\infty} \int_0^\infty \frac{dx}{1+x+x^2n^{-2}} \qquad 4. \lim_{n\to\infty} \int_0^\infty \frac{\operatorname{arctg}(nx)}{1+x^3}\,dx$$

$$5. \lim_{n\to\infty} \int_1^\infty \frac{dx}{\ln x + \ln n} \qquad 6. \lim_{n\to\infty} \int_0^\infty \frac{x^n}{1+x^{2n}}\,dx$$

$$7. \lim_{n\to\infty} \int_0^\infty \frac{\ln(x+n)}{n}\exp(-x)\cos x\,dx \qquad 8. \lim_{n\to\infty} \int_0^{23} \frac{\exp(x^3)}{1+n\sqrt{x}}\,dx$$

C. Rozviňte v řadu:

$$1. \int_0^1 \ln\left(\frac{1+x}{1-x}\right)dx \qquad 2. \int_0^\infty \exp(-x)\cos\sqrt{x}\,dx \qquad 3. \int_0^1 \ln(x)\ln(1-x)\,dx$$

$$4. \int_0^1 \frac{x^p\ln x}{1+x^2}\,dx \qquad 5. \int_0^\infty \frac{\sin x}{1+\exp x}\,dx \qquad 6. \int_0^\infty \frac{x}{\exp x - 1}\,dx$$

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D. Vyšetřete spojitost funkce $F(a)$:

$$1. \int_0^\infty \frac{\exp(-ax)}{1 + x^3}\, dx, \quad a \geq 0 \qquad 2. \int_0^\infty \frac{x}{1 + x^a}\, dx, \quad a > 1$$

$$3. \int_0^\pi \frac{\sin x}{x^a(\pi - x)^a}\, dx, \quad 0 < a < 2 \qquad 4. \int_{-1}^1 \sqrt{x^2 + a^2}, \quad a \in R$$

$$5. \int_0^\infty \exp(-ax)\frac{\sin x}{x}\, dx, \quad a > 0 \qquad 6. \int_1^\infty \frac{\sin\left(\frac{1}{x}\right)}{x(a + x)^2}, \quad a > -1$$

E. Derivováním podle parametru spočtěte integrály:

$$1. \int_0^\infty \frac{1 - \exp(-ax^2)}{x^2 \exp(x^2)}\, dx \qquad 2. \int_0^1 \frac{x^a - 1}{\ln x}\, dx$$

$$3. \int_0^\infty \exp(-x)\frac{\sin(ax)}{x}\, dx \qquad 4. \int_0^\infty \exp(-x)\frac{1 - \cos(ax)}{x}\, dx$$

$$5. \int_0^\infty \frac{\operatorname{arctg}(ax)}{x(1 + x^2)}\, dx \qquad *6. \int_0^\infty \exp\left\{-\left(x + \frac{a}{x}\right)\right\}\, dx$$

F. Vyšetřete průběh funkce $F(a)$:

$$1.\ F(a) = \int_0^\infty \frac{\exp(-ax)}{1 + x^2}\, dx \qquad 2.\ F(a) = \int_0^1 \frac{\ln(1 - a^2 x^2)}{x^2\sqrt{1 - x^2}}\, dx$$

$$3.\ F(a) = \int_0^1 \frac{dx}{\sqrt{x^2 + a^2}} \qquad 4.\ F(a) = \int_1^\infty \frac{\operatorname{arctg}(ax)}{x^2\sqrt{x^2 - 1}}\, dx$$