A. Naleznte Fourierovu transformaci funkc:

1. $f(x) = \chi_{(-1,1)}(x)$ a s jej pomoc $f(x) = \chi_{(a,b)}(x)$, $a < b$.

2. $f(x) = \chi_{(0,1/2)}(x) - \chi_{(1/2,1)}(x)$

3. $f(x) = (1+x)\chi_{(-1,0)}(x) + (1+x)\chi_{(0,1)}(x)$

4. $f(x) = \chi_{(-\pi,\pi)}(x)\sin x$

5. $f(x) = \chi_{(-\pi/2,\pi/2)}(x)\cos x$

6. $f(x) = x\exp(-ax^2)$. (Uijte faktu $\mathcal{F}[\exp(-\pi x^2)](\xi) = \exp(-\pi\xi^2)$).

(Pozn.: $\chi_{(a,b)}(x)$ je charakteristick funkce intervalu (a,b).)

B. Naleznte Fourierovu transformaci funkc:

1. $f(x) = \frac{1}{x^2+i}$

2. $f(x) = \frac{1}{x^2+x+1}$

3. $f(x) = \frac{1}{x+i}$ a potamo $f(x) = \frac{1}{(x+i)^n}$

4. $f(x) = \frac{x}{(x-i)^2}$

5. $f(x) = \frac{1}{e^x+e^{-x}}$

6. $f(x) = \frac{1}{e^x+e^{-x}+2}$

Nvod: integrujte funkci $\exp(2\pi i \xi z)f(z)$ pes:
1–4: horn a doln polokrunici o stedu 0 a polomru R,
5,6: obdlnk s vrcholy $R, R+2\pi i, -R+2\pi i, -R$.
Uijte reziduovou vtu a limitn pechod $R \to \infty$.

C. Naleznte Laplaceovou transformaci funkc:

1. $f(t) = t\chi_{(0,1)}(t)$

2. $f(t) = \chi_{(0,A)}(t)$

3. $f(t) = t^k \sin(at)$

4. $f(t) = t^k \cos(at)$

5. $f(t) = t^k \cosh(at)$

6. $f(t) = t^k \sinh(at)$

D. Naleznte inverzn Laplaceovu transformaci funkc:

1. $F(p) = \frac{1}{(p+a)^2+b^2}$

2. $F(p) = \frac{1}{(p-a)^2-b^2}$

3. $F(p) = \frac{p+1}{p^2-2p+1}$

4. $F(p) = \frac{1}{p^2+2p+1}$

5. $F(p) = \frac{1}{p^2+p+1}$

6. $F(p) = \frac{1}{p^2+4p+9}$

E. Pomoc Laplaceovy transformace ete rovnice:

1. $x' - 2x = \sin(at)$, $x(0) = 0$.

2. $x' + x = 1$, $x(0) = 1$.

3. $x'' - 3x' + 2x = 0$, $x(0) = 0$, $x'(1) = 1$.

4. $x'' + 6x' - 7x = 0$, $x(0) = 1$, $x'(1) = 0$.

5. $x(t) = t + \int_0^t x(s) \cdot (t - s)\, ds$

6. $x(t) = \cos t + \int_0^t x(s) \sin(t - s)\, ds$