

1. Helmholtz decomposition

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Exercise 1.

Consider following velocity vectors:

$$\mathbf{u}_1 = (y, 0, 0), \mathbf{u}_2 = (-y, x, 0), \mathbf{u}_3 = \left(-\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, 0 \right)$$

Plot the velocity fields and compute their curl. In which sense does it hold that in the rotating flow, the velocity curl is non-zero?

Exercise 2.

Consider vector fields from Exercise 1.

a) Are these fields divergent? b) If it holds that the curl of the given vector field is identically zero in \mathbb{R}^3 , can we deduce something about its divergence? (Think about the Helmholtz decomposition.)

Exercise 3.

For the velocity field $\mathbf{u} = (-y, x, 0)$, find an arbitrary scalar and vector potential ϕ and \mathbf{A} , so that

$$\mathbf{u} = \nabla\phi + \nabla \times \mathbf{A} \tag{1}$$

inside a bounded domain in \mathbb{R}^3 .