# 11. Coriolis force

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**Problem 1.**
Check the validity of the phrase ”apple doesn’t fall far from the tree”for an apple tree at the equator after considering the Coriolis force. Consider that the apple falls from the height $h = 4$ m. Neglect the air resistance as well as the effect of the horizontal velocity caused by the Coriolis force.

**Solution:**

On every point on the Earth, we consider a local Cartesian system, where axis $x$ points towards the east, axis $y$ towards the north and axis $z$ vertically upwards. In this system, the angular velocity vector of the Earth rotation is

$$\mathbf{\Omega} = (0, \Omega \cos\varphi, \Omega \sin\varphi)\,,$$

where $\Omega = 2\pi/(24 \text{ hours})$ is the size of the angular velocity and $\varphi$ is the latitude. We will first consider general velocity $\mathbf{u} = (u, v, w)$. Coriolis force for unit mass can be expressed by formula

$$\mathbf{f}_{\mathbf{Cor}} = -2\mathbf{\Omega} \times \mathbf{u} = \begin{pmatrix} -2\Omega\,(w\cos\varphi - v\sin\varphi) \\ -2\Omega u \sin\varphi \\ 2\Omega u \cos\varphi \end{pmatrix}.$$

Using the relation for the Coriolis force, Coriolis force causes the acceleration of the falling apple only in the direction $x$, which is

$$a_x = -2\Omega w \cos\varphi.$$

In the vertical direction, neglecting the air resistence, it is therefore uniformly accelerated motion. Because the apple is initially at rest, its vertical velocity is described by equation $w = -gt$ and a further integration, we would get formula for the position of the apple

$$z = h - \frac{1}{2}gt^2.$$

The apple will therefore fall to the ground (distance $h$) after time

$$t = \sqrt{\frac{2h}{g}}.$$

All the time, the apple is accelerating in the direction $x$ by the Coriolis force. The corresponding velocity can be obtained by integration of the acceleration as

$$u = \Omega g t^2 \cos\varphi.$$

It will therefore (in the eastward direction) cover the path

$$x = \frac{1}{3}\Omega g t^3 \cos\varphi = \frac{1}{3}\Omega g \left(\frac{2h}{g}\right)^{\frac{3}{2}} \cos\varphi.$$

After substituting the numbers for the equator and the height 4 m, it is approximately

$$x \approx \frac{1}{3} 7.3e^{-5} 10 \left(\frac{2*4}{10}\right)^{\frac{3}{2}} \cos 0 \approx 0.17\,\text{mm}.$$

In the Czech republic, it is only 0.12 mm.

**Problem 2.**
Look at a particle moving from west to east on the northern hemisphere, that was dislocated to a more poleward position by some external perturbation. What will happen then, if we consider the effect of the Coriolis force in the form $\mathbf{F} = (fv, -fu, 0)$, where $f = 2\Omega \sin\varphi$ for the latitude $\varphi$? Consider barotropic (density is a function of pressure), horizontal and non-divergent flow, for which the vorticity equation takes the form $\mathrm{d}(\zeta + f)/\,\mathrm{d}t = 0$, where $\zeta$ is the relative vorticity.

**Solution:**
First, we notice that the Coriolis parameter $f$ is 0 at the equator and increases towards the pole.

Second, we note that the Coriolis form in this form is perpendicular to the velocity ($\mathbf{F} \cdot \mathbf{v} = 0$) and for positive $f$ (Northern hemisphere) it points to the right from the moving particle (for example, when considering the velocity $(1, 0, 0)$ – corresponding Coriolis force would be $(0, -f, 0)$).

The particle that was displaced on the northern hemisphere towards north was displaced towards the pole, the Coriolis effect will therefore increase. From the vorticity equation, this means that the relative vorticity decreases. This implies that the particle will have larger tendency to rotate in the negative (anticyclonic, clockwise) direction. It will therfore start a circular motion. After it goes to the furthest point, the parameter $f$ will however start decreasing and the vorticity would be decreasing, too (it will be still anticyclonic, only smaller).

After some time, it will reach the original latitude. However, due to the inertia, it would create a perturbation to the south. Again, this has effect on the decreasing Coriolis parameter, and there will be increasing positive vorticity (cyclonic, counter-clockwise). It will therefore start a circular motion to the opposite direction than before.

This way, the particle will be following a sinusoidal trajectory. This effect is one of the basic motion types in the atmosphere, called Rossby waves.