3. Material derivative

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Problem 1.
Consider flow that is described in a lagrangian way using equations

$$x = Xe^{\alpha t}, \quad y = Ye^{-\alpha t}, \quad z = Z,$$

where X, Y and Z are the initial position of a fluid parcel and $\alpha > 0$. During the last tutorial, we derived for this flow that it holds, among others,

$$u(X,Y,Z,t) = \alpha X e^{\alpha t}, \quad v(X,Y,Z,t) = -\alpha Y e^{-\alpha t}, \quad w(X,Y,Z,t) = 0,$$
$$u(x,y,z,t) = \alpha x, \quad v(x,y,z,t) = -\alpha y, \quad w(x,y,z,t) = 0.$$

- Let the concentration of a pollutant be defined as $c(x,y,t) = \beta x^2 y e^{-\alpha t}$. Does the concentration change in time for a fluid parcel?
- Evaluate the acceleration in the direction x using both lagrangian and eulerian description.

Problem 2.
Decide when the following properties of the material derivatives $\mathrm{D}/\,\mathrm{D}t$ hold:
a) Formula for the derivative of a sum of two arbitrary smooth functions f, g:

$$\frac{\mathrm{D}}{\mathrm{D}t}(f+g) = \frac{\mathrm{D}}{\mathrm{D}t}f + \frac{\mathrm{D}}{\mathrm{D}t}g.$$

b) Formula for the derivative of product of two arbitrary smooth funcntions f, g:

$$\frac{\mathrm{D}}{\mathrm{D}t}(fg) = f\frac{\mathrm{D}}{\mathrm{D}t}g + g\frac{\mathrm{D}}{\mathrm{D}t}f.$$

c) Exchange of order of the material derivative and the partial spatial derivative of an arbitrary smooth function f:

$$\frac{\mathrm{D}}{\mathrm{D}t}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\mathrm{D}f}{\mathrm{D}t}.$$