4. Inertial oscillation

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Problem 1.
Inertial oscillation is a special type of motion of the air in the atmosphere, in which the inertia of the fluid is balanced by the Coriolis force.
a) This fluid can be described in the eulerian description by equations

$$\frac{\partial u}{\partial t} - fv = 0,$$
$$\frac{\partial v}{\partial t} + fu = 0,$$

where f is the Coriolis parameter, taken as a constant here. Find the period of the oscillations.
b) In the lagrangean description, one can write (outside the equatorial region) equations for motion of an air parcel

$$-K_H|\mathbf{u}|\mathbf{u} \times \mathbf{k} = f\mathbf{u} \times \mathbf{k},$$

where K_H is the horizontal curvature of the motion and $\mathbf{k}$ is a vector pointing upwards in the direction of the z-axis. The equation describes the equality of the centrifugal force created due to the horizontal curvature of the streamlines and the Coriolis force. How does this motion look like in the middle latitudes ($f \approx 10^{-4}$ s^{-1}) with the flow velocity 10 m/s, if the Coriolis parameter is constant? How does the trajectory change, if we consider the dependence of the Coriolis parameter on the latitude $f = 2\Omega \sin(\varphi)$ (Ω is the angular frequency for the rotation of the Earth)?