

5. Fluid dynamics

15. November 2023

Problem 1.

Consider two-dimensional flow described by lagrangian equations

$$x = Xe^{-at}, y = Y + bt,$$

where X and Y specify the original position and a and b are positive constants. Check that the lagrangian and eulerian acceleration coincide.

Problem 2.

Stress in Newtonian (Navier-Stokes) equation can be described by the formula

$$\mathbb{T} = -p\mathbb{I} + 2\mu\mathbb{D},$$

where \mathbb{D} is the symmetric part of the velocity gradient defined as $\mathbb{D} = (D_{ij})_{i,j=1,2,3}$, $D_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$. Show that for the simple shear flow $\mathbf{u} = (u(y), 0, 0)$, all diagonal components of the stress tensor \mathbb{T} are the same. Consider further the Stokes fluid, for which we have (instead of the previous formula)

$$\mathbb{T} = -p\mathbb{I} + \alpha_1\mathbb{D} + \alpha_2\mathbb{D}^2.$$

How does the tensor \mathbb{T} look like this time? Can this have some consequences for the flow? (The equations of motion in this formulation are written as $\rho \mathbf{du}/dt = \nabla \cdot \mathbb{T} + \rho \mathbf{b}$, where \mathbf{b} is an external force.)