Hydrodynamics
Homework 7: Circulation
29. November 2023

Problem:
Consider the flow

$$\mathbf{u} = \left(1 + \cos\left(\frac{s}{2}\right), \frac{1}{2}\sin s, 0\right),$$

where the real parameter s refers to the initial position of the particles. Further consider three different time-dependent curves

$$\mathbf{x}_1 = \left(t\left(1 + \cos\left(\frac{s}{2}\right)\right), \frac{t}{2}\sin s, 0\right), \quad 0 \leq s < 2\pi,$$
$$\mathbf{x}_2 = \left(t\left(1 + \cos\left(\frac{s}{2}\right)\right), \frac{t}{2}\sin s, 0\right), \quad 0 \leq s < 4\pi,$$
$$\mathbf{x}_3 = \left(t\left(1 + \cos s\right), \frac{t}{2}\sin s, 0\right), \quad 0 \leq s < 2\pi.$$

For which of these curves does the Kelvin circulation theorem imply that the circulation

$$\Gamma_i = \int_{\mathbf{x}_i(t)} \mathbf{u} \cdot \mathrm{d}\mathbf{x}, \quad i = 1, 2, 3$$

does not depend on time? You can verify your choice by direct computation.