Hydrodynamics
Homework 8: Hamiltonian mechanics
6. December 2023

Problem:
Consider the Hamiltonian description of the rigid body motion with the Poisson bracket defined by the bivector $L^{ij} = -m_k \varepsilon_{kij}$ with ε being the Levi-Civita symbol and $\mathbf{m}$ the angular momentum describing the phase space. Substituting the Hamiltonian

$$H(\mathbf{m}) = \frac{1}{2}\left(\frac{m_1^2}{I_1} + \frac{m_2^2}{I_2} + \frac{m_3^2}{I_3}\right),$$

into the general equation $\dot{\mathbf{m}} = \{\mathbf{m}, H\}$ would lead to the equations of motion in the form

$$\dot{\mathbf{m}} = \mathbf{m} \times \boldsymbol{\Omega},$$

where $\Omega_i = m_i/I_i$ is the angular velocity.

How would the equations of motion change, if we start with a different Hamiltonian

$$H(\mathbf{m}) = \frac{1}{2}\left(\frac{m_1^2(1+I_1)}{I_1} + \frac{m_2^2(1+I_2)}{I_2} + \frac{m_3^2(1+I_3)}{I_3}\right)$$

instead of the original one?

(One-line answer might be the correct answer - no long computations needed. Results from the tutorials can help.)