Hydrodynamics
Homework 9: Vorticity
13. December 2023

Problem:
Consider the closed material curve

$$\mathbf{x} = (t\cos s, t\sin s, 0), \quad 0 \leq s < 2\pi$$

and the corresponding flow velocity

$$\mathbf{u} = \frac{\partial \mathbf{x}}{\partial t} = (\cos s, \sin s, 0) = \left(\frac{x}{t}, \frac{y}{t}, 0\right).$$

Calculate the circulation using both formula

$$\Gamma = \int_{\mathbf{x}(t)} \mathbf{u} \cdot d\mathbf{x}$$

and

$$\Gamma = \int_{\text{Int}(\mathbf{x}(t))} \omega \, dx \, dy,$$

where ω is the vorticity and $\text{Int}(\mathbf{x}(t))$ denotes the area formed by the curve. Are the results the same?