9. Vorticity

13. December 2023

Problem 1.
Consider the following two examples of incompressible inviscid two-dimensional flow with constant density ρ, described by the velocities

$$\mathbf{u}_1 = (2Ay, -2Ax), \quad \mathbf{u}_2 = (\frac{Ay}{x^2+y^2}, -\frac{Ax}{x^2+y^2})$$

Find the vorticity for both the flows. If the flow is irrotational, find also the pressure, assuming that the gravity is negligible and the pressure for $r \to \infty$ equals a known value p_∞.

Problem 2.
To the flow defined by the motion of particles on a curve $C(t)$,

$$\mathbf{x} = (a\cos s + a\alpha t \sin s, a \sin s, 0), \quad 0 \leq s < 2\pi,$$

for which we obtained the velocity $u = (\alpha y, 0, 0)$ and the circulation $\Gamma = -\alpha a^2 \pi$ at the Tutorial 7, compute the vorticity. For time $t = 0$, integrate the vorticity over the area inside the curve $C(t)$. Why is the result same as the circulation? How will the area encompassed by the curve $C(t)$ evolve for time $t > 0$?