1. Helmholtz decomposition

3. October 2024

Exercise 1.
Consider following velocity vectors:

$$\mathbf{u}_1 = (y, 0, 0), \mathbf{u}_2 = (-y, x, 0), \mathbf{u}_3 = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right)$$

Plot the velocity fields and compute their curl. In which sense does it hold that in the rotating flow, the velocity curl is non-zero?

Exercise 2.
Consider vector fields from Exercise 1.
a) Are these fields divergent? b) If it holds that the curl of the given vector field is identically zero in $\mathbb{R}^3$, can we deduce something about its divergence? (Think about the Helmholtz decomposition.)

Exercise 3.
For the velocity field $\mathbf{u} = (-y, x, 0)$, find an arbitrary scalar and vector potential ϕ and $\mathbf{A}$, so that

$$\mathbf{u} = \nabla\phi + \nabla \times \mathbf{A} \tag{1}$$

inside a bounded domain in $\mathbb{R}^3$.