# 3. Trajectory and streamline

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**Problem 1.**
Consider flow that is described in a lagrangian way using equations

$$x = Xe^{\alpha t}, \quad y = Ye^{-\alpha t}, \quad z = Z,$$

where $X$, $Y$ and $Z$ are the initial position of a fluid parcel and $\alpha > 0$.

- Find its trajectory.
- Find and plot its streamlines.
- Decide whether the flow is stationary.
- Let the concentration of a pollutant be defined as $c(x, y, t) = \beta x^2 y e^{-\alpha t}$. Does the concentration change in time for a fluid parcel?
- Evaluate the acceleration in the direction $x$ using both lagrangian and eulerian description.

**Problem 2.**
Consider the nonstationary flow

$$u = u_0, \quad v = kt, \quad , w = 0,$$

where $u_0$ and $k$ are positive constants. Derive how the streamlines look like and find the trajectories.

**Problem 3.**
Decide when the following properties of the material derivatives $\mathrm{D}/\mathrm{D}t$ hold:
a) Formula for the derivative of a sum of two arbitrary smooth functions $f$, $g$:

$$\frac{\mathrm{D}}{\mathrm{D}t}(f + g) = \frac{\mathrm{D}}{\mathrm{D}t}f + \frac{\mathrm{D}}{\mathrm{D}t}g.$$

b) Formula for the derivative of product of two arbitrary smooth funcntions $f$, $g$:

$$\frac{\mathrm{D}}{\mathrm{D}t}(fg) = f\frac{\mathrm{D}}{\mathrm{D}t}g + g\frac{\mathrm{D}}{\mathrm{D}t}f.$$

c) Exchange of order of the material derivative and the partial spatial derivative of an arbitrary smooth function $f$:

$$\frac{\mathrm{D}}{\mathrm{D}t}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\mathrm{D}f}{\mathrm{D}t}.$$