4. Incompressibility

31. October 2024

Problem 1.
Consider two-dimensional flow described by lagrangian equations

$$x = Xe^{-at}, y = Y + bt,$$

where X and Y specify the original position and a and b are positive constants. Check that the lagrangian and eulerian acceleration coincide.

Problem 2.
Let us have a general velocity field $\mathbf{u}$ with the density ρ. Consider a volume $\mathcal{V}(t)$ inside the fluid in time t composed of particles that take the volume $V(t_0)$ at the initial time t_0. How can be expressed the fact that the field is incompressible? What does it mean for the velocity? What does it mean for the density?

Problem 3.
Consider the flow with velocity $\mathbf{u} = (x, y, 0)$. Is this flow incompressible?