

Hydrodynamics  
Homework 5: Fluid dynamics  
7. November 2024

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**Exercise:**

Consider the power-law fluid model defined by

$$\mathbb{T} = -p\mathbb{I} + 2\mu_0\|\mathbb{D}\|^{p-2}\mathbb{D}$$

with the notation from the tutorials and constants  $p$  and  $\mu_0$ . This model describes a fluid with modified viscosity compared to the standard incompressible Navier Stokes model

$$\mathbb{T} = -p\mathbb{I} + 2\mu\mathbb{D}.$$

Express, in which way is the viscosity modified for the simple shear flow  $\mathbf{u} = (u(y), 0, 0)$ . That is, express the second term in the stress tensor  $\mathbb{T}$  for both models and compare them.

Fluids, for which the viscosity increases with increased shear ( $du/dy$ ), are called shear thickening fluids - the fluids run better if there is pressure. This is a useful property for example for paints. Fluids with decreasing viscosity with increased shear are called shear thinning fluids.

The matrix norm  $\|\mathbb{D}\|$  of a matrix with components  $D_{ij}$  can be computed as

$$\|\mathbb{D}\| = \sqrt{\sum_{ij} D_{ij}^2}$$