5. Fluid dynamics

7. November 2024

Problem 1.
Stress in Newtonian (Navier-Stokes) equation can be described by the formula

$$\mathbb{T} = -p\mathbb{I} + 2\mu\mathbb{D},$$

where $\mathbb{D}$ is the symmetric part of the velocity gradient defined as $\mathbb{D} = (D_{ij})_{i,j=1,2,3}$, $D_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$. Show that for the simple shear flow $\mathbf{u} = (u(y), 0, 0)$, all diagonal components of the stress tensor $\mathbb{T}$ are the same. Consider further the Stokes fluid, for which we have (instead of the previous formula)

$$\mathbb{T} = -p\mathbb{I} + \alpha_1\mathbb{D} + \alpha_2\mathbb{D}^2.$$

How does the tensor $\mathbb{T}$ look like this time? Can this have some consequences for the flow? (The equations of motion in this formulation are written as $\rho\,\mathrm{d}\mathbf{u}/\,\mathrm{d}t = \nabla\cdot\mathbb{T} + \rho\mathbf{b}$, where $\mathbf{b}$ is an external force.)

Problem 2.
Decide, if the parity described by the following equations

$$\mathbf{r}' = -\mathbf{r},$$
$$t' = t,$$
$$p' = p,$$

is a symmetry for the Euler equations containing the gravity

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p - g\mathbf{k},$$

where $\mathbf{k}$ is a unit vector in the direction z. If not, how do we have to modify the transformation, so that it is a symmetry?