Hydrodynamics
Homework 6: Circulation
14. November 2024

Problem:
Consider the flow

$$\mathbf{u} = \left(1 + \cos\left(\frac{s}{2}\right), \frac{1}{2}\sin s, 0\right),$$

where the real parameter s refers to the initial position of the particles. Further consider three different time-dependent curves

$$\mathbf{x}_1 = \left(t\left(1 + \cos\left(\frac{s}{2}\right)\right), \frac{t}{2}\sin s, 0\right), \quad 0 \leq s < 2\pi,$$
$$\mathbf{x}_2 = \left(t\left(1 + \cos\left(\frac{s}{2}\right)\right), \frac{t}{2}\sin s, 0\right), \quad 0 \leq s < 4\pi,$$
$$\mathbf{x}_3 = \left(t\left(1 + \cos s\right), \frac{t}{2}\sin s, 0\right), \quad 0 \leq s < 2\pi.$$

For which of these curves does the Kelvin circulation theorem imply that the circulation

$$\Gamma_i = \int_{\mathbf{x}_i(t)} \mathbf{u} \cdot \mathrm{d}\mathbf{x}, \quad i = 1, 2, 3$$

does not depend on time? You can either check the assumptions of the theorem or verify your choice by direct computation. Feel free to use WolframAlpha or similar software to simplify the integrations.