6. Circulation

14. November 2024

Problem 1.
Consider the curve $C(t)$ given by the particles of the fluid

$$\mathbf{x} = (a\cos s + a\alpha t \sin s, a\sin s, 0), \quad 0 \leq s < 2\pi.$$

By the direct computation, show that

$$\Gamma = \int_{C(t)} \mathbf{u} \cdot \mathrm{d}\mathbf{x} = \int_0^{2\pi} \mathbf{u} \cdot \frac{\partial \mathbf{x}}{\partial s} \, \mathrm{d}s$$

does not depend on time. Why?

Problem 2.
Compute the vorticity for the flow in the previous problem. Integrate the vorticity for time $t = 0$ over the area defined by the curve $C(t)$. Explain the result. How will the area enclosed by the curve $C(t)$ evolve in time $t > 0$?

Problem 3.
Consider an inviscid, incompressible flow in a cylindrical pipe depicted on Fig. 1. Velocity in the zone 1 is described as $\mathbf{u}_1 = (0, 0, u_1)$ with

$$u_1(r) = U_1 \left(1 - \left(\frac{r}{R_1}\right)^2\right). \tag{1}$$

Compute the vorticity and the velocity in the zone 2.

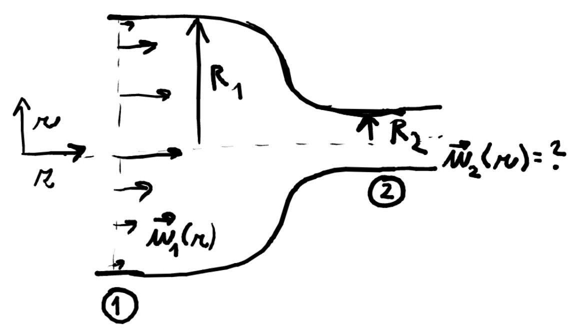

Figure 1: Scheme for the Problem 3.