9. Thermodynamics and waves

12. December 2024

Problem 1.
The atmosphere of Mars is composed mainly of CO_2, which has the specific heat at constant pressure $c_p^m = 844$ J/kg/K and the specific heat at constant volume $c_v^m = 655$ J/kg/K. The gravity on the surface is 3.7 m/s^{-2} and the temperature in the summer near the equator is around 20°C.

- Compute the adiabatic lapse rate and compare it with the lapse rate on the Earth.
- Compute the speed of sound and compare it with the speed of sound on the Earth.

The specific heats for the Earth are $c_p = 1005$ J/kg/K and $c_v = 718$ J/kg/K.

Problem 2.
Look at a particle moving from east to west on the northern hemisphere, that was dislocated to a more poleward position by some external perturbation. What will happen then, if we consider the effect of the Coriolis force in the form $\mathbf{F} = (fv, -fu, 0)$, where $f = 2\Omega \sin\varphi$ for the latitude φ? Consider barotropic (density is a function of pressure), horizontal and non-divergent flow, for which the vorticity equation takes the form $\mathrm{d}(\zeta + f)/\,\mathrm{d}t = 0$, where ζ is the relative vorticity.

Problem 3.
Equation for geostrophic equilibrium with friction can be written in the form

$$\mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_0}\nabla p + \frac{1}{\rho_0}\frac{\partial \boldsymbol{\tau}}{\partial z}$$

where $\mathbf{f} = (f, -f)$ and $\boldsymbol{\tau}$ is the stress vector. We are considering the Boussinesq aproximation, in which the density is divided into the constant density ρ_0 and a small perturbation.

Over the tropical ocean, we can observe atmospheric cyclons. Compute the effect of such a storm on the Ekman layer in the infinitely deep ocean.

The stress of air caused by a cyclonic storm can be described as

$$\boldsymbol{\tau} = -A \exp^{-(r/\lambda)^2}(y, x, 0),$$

where $r^2 = x^2 + y^2$ and A and λ are constants. The ocean is further affected by the turbulence inside the ocean flow $\boldsymbol{\tau}_t = \rho_0 K \partial_z \mathbf{u}$, where K is a constant. Assume that the flow can be divided into a homogeneous geostrophic component $(\bar{u}, \bar{v}, 0)$ and a perturbation corresponding to the Ekman layer.