10. Coriolis force 2

11. December 2025

Problem 1.
Consider air moving toward the north with the velocity $v = 10$ m/s at 45°N. How much will it deviate eastward within an hour?

Problem 2.
Inertial oscillation is a special type of motion of the air in the atmosphere, in which the inertia of the fluid is balanced by the Coriolis force.
a) This fluid can be described in the eulerian description by equations

$$\frac{\partial u}{\partial t} - fv = 0,$$
$$\frac{\partial v}{\partial t} + fu = 0,$$

where f is the Coriolis parameter, taken as a constant here. Find the period of the oscillations.
b) In the lagrangean description, one can write (outside the equatorial region) equations for motion of an air parcel

$$-K_H|\mathbf{u}|\mathbf{u} \times \mathbf{k} = f\mathbf{u} \times \mathbf{k},$$

where K_H is the horizontal curvature of the motion and $\mathbf{k}$ is a vector pointing upwards in the direction of the z-axis. The equation describes the equality of the centrifugal force created due to the horizontal curvature of the streamlines and the Coriolis force. How does this motion look like in the middle latitudes ($f \approx 10^{-4}$ s^{-1}) with the flow velocity 10 m/s, if the Coriolis parameter is constant? How does the trajectory change, if we consider the dependence of the Coriolis parameter on the latitude $f = 2\Omega \sin(\varphi)$ (Ω is the angular frequency for the rotation of the Earth)?

Problem 3.
Stress in Newtonian (Navier-Stokes) equation can be described by the formula

$$\mathbb{T} = -p\mathbb{I} + 2\mu\mathbb{D},$$

where $\mathbb{D}$ is the symmetric part of the velocity gradient defined as $\mathbb{D} = (D_{ij})_{i,j=1,2,3}$, $D_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$. Show that for the simple shear flow $\mathbf{u} = (u(y), 0, 0)$, all diagonal components of the stress tensor $\mathbb{T}$ are the same. Consider further the Stokes fluid, for which we have (instead of the previous formula)

$$\mathbb{T} = -p\mathbb{I} + \alpha_1\mathbb{D} + \alpha_2\mathbb{D}^2.$$

How does the tensor $\mathbb{T}$ look like this time? Can this have some consequences for the flow? (The equations of motion in this formulation are written as $\rho\,\mathrm{d}\mathbf{u}/\,\mathrm{d}t = \nabla \cdot \mathbb{T} + \rho\mathbf{b}$, where $\mathbf{b}$ is an external force.)