8. Stream function

20. November 2025

Problem 1.
Decide, if the parity described by the following equations

$$\mathbf{r}' = -\mathbf{r},$$
$$t' = t,$$
$$p' = p,$$

is a symmetry for the Euler equations containing the gravity

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p - g\mathbf{k},$$

where $\mathbf{k}$ is a unit vector in the direction z. If not, how do we have to modify the transformation, so that it is a symmetry?

Problem 2.
Find the velocities for the following stream functions. Test whether the velocities correspond to a potential flow and the fields are incompressible.

$$\psi_1 = Axy, \quad \psi_2 = A(x^2 - y^2).$$

Problem 3.
For the following fields, find the stream function and the velocity potential:

a) Couette flow: Flow between two infinitely long horizontal plates with distance h. One of them is moving with velocity U and the second one is stationary. The velocity of the Couette flow is

$$u = U\frac{y}{h}, \quad v = 0.$$

b) Velocity field

$$u = A(x^2 - y^2), \quad v = -2Axy,$$

where A is a constant.