Modeling of porous flow with strong mechanical coupling in planetary applications

Ondřej Souček[1], Klára Kalousová[2] Gabriel Tobie[3], Gaël Choblet[3], and Ondřej Čadek[2]

[1]Faculty of Mathematics and Physics, Charles University in Prague, Mathematical Institute, Czech Republic,
[2]Faculty of Mathematics and Physics, Charles University in Prague, Department of Geophysics, Czech Republic,
[3]Laboratoire de Planétologie et Géodynamique, Université de Nantes, France.

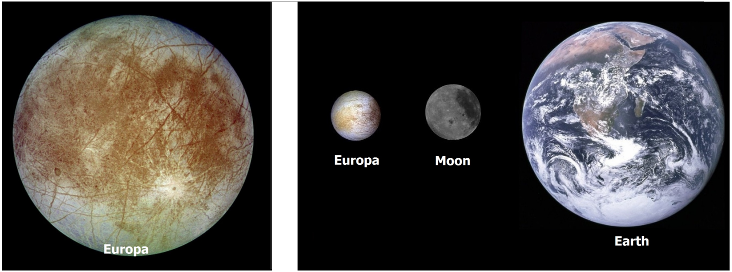
29.4.2015

Outline

1. Motivation: Europa
   - Structure, orbital & interior dynamics, surface composition & age
   - Surface geology
   - Shallow liquid water, melting processes
2. Two-phase flow model + extensions
3. Numerical simulations
   - Sensitivity study (1d)
   - Fully temperate case (2d)
   - Europa: Water transport by two-phase flow (1d)
   - Europa: Impermeable limit - water transport by ice advection (2d)
4. Conclusions and perspectives

Europa: Interior structure

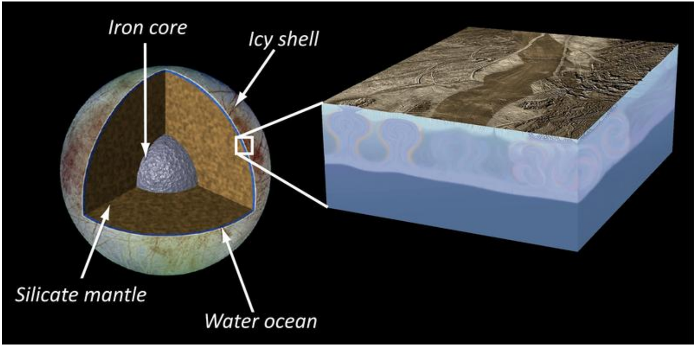
- Smallest of the Galilean satellites of Jupiter ($R = 0.243 R_{Earth}$)
- Gravity data → metal core, silicate mantle, outer water-ice layer (*Anderson et al.*, 1998)
- Magnetic data → global subsurface ocean ($\sim 100 km$) + thin ice shell (*Kivelson et al.*, 2000)
- Ice shell thickness from ≤10 km to ≥40 km (*Billings & Kattenhorn*, 2005)



Courtesy NASA/JPL-Caltech

Europa: Interior structure

- Smallest of the Galilean satellites of Jupiter ($R = 0.243 R_{Earth}$)
- Gravity data $\rightarrow$ metal core, silicate mantle, outer water-ice layer (*Anderson et al.*, 1998)
- Magnetic data $\rightarrow$ global subsurface ocean ($\sim 100 km$) + thin ice shell (*Kivelson et al.*, 2000)
- Ice shell thickness from ≤10 km to ≥40 km (*Billings & Kattenhorn*, 2005)



Courtesy NASA/JPL-Caltech

Europa: Dynamics, Surface composition & Age

- In Laplace resonance with Io and Ganymede → non-zero eccentricity
- Eccentric orbit → significant tidal deformations (*Showman & Malhotra*, 1997) and heating (*Tobie et al.*, 2003), possibly several times larger than radiogenic heating in the rocky core (*Sotin et al.*, 2009)
- Dearth of impact craters → Very young surface ∼40−90 Myr (*Bierhaus et al.*, 2009) → Ongoing geological activity?
- Water vapor plumes above Europa's south pole (*Roth et al.*, 2014) → Liquid water at shallow depth? Ongoing interior activity?

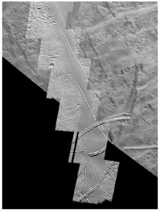
Europa: Surface geology - abundance of unique features

Tectonic features, Chaotic Terrain,...:

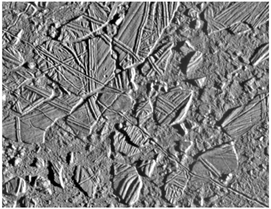
Double ridges

Cycloids

Strike-slip faults



Chaotic Terrain

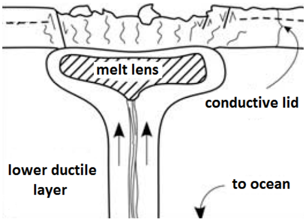


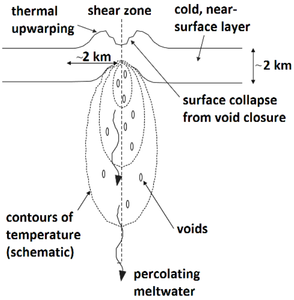
Lenticulae

Courtesy NASA/JPL-Caltech

Europa: Melting processes

- In hot plumes (*Sotin et al.*, 2002)
- melting is a result of tidal heating enhanced due to thermally-reduced viscosity
- At strike-slip faults (*Nimmo & Gaidos*, 2002)
- melting as shallow as few km can initiate for shear velocities appropriate for Europa's diurnal tides



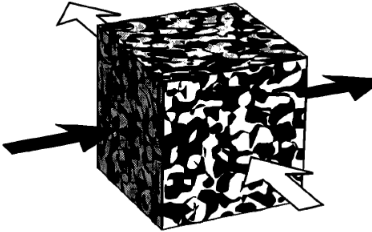


Water transport mechanisms in Europa's ice shell

- Crevasse hydrofracturing:
  - crack propagation promoted by meltwater supply
  - dominant on the Earth, rapid water drainage (*Krawczynski et al.*, 2009)
- Rayleigh-Taylor instability
  - if no cracks/pores (impermeable ice) → collapse of gravitationally unstable partially molten ice
- **Two-phase flow**:
  - if no fractures → meltwater flow through the shell compensated by the ice flow → mechanical coupling between the phases
  - ∼ silicate magma generation + transport through the Earth's mantle

Two-phase flow: multi-phase theory

- Single-component balances on meso-scopic subdomains
- Transition conditions at interfaces
- Averaging over representative meso-scale volume $\rightarrow$ continuum description formally similar to mixture theory



from Bercovici et al., 2001

Binary mixture - balance laws

Traditional terms component-wise + novel (interaction) terms

- Balances of mass (for individual components)

$$\frac{\partial(\phi\varrho_f)}{\partial t} + \operatorname{div}(\phi\varrho_f \boldsymbol{v}_f) = \underbrace{r_f}_{\text{melt rate}} ,$$

$$\frac{\partial((1-\phi)\varrho_m)}{\partial t} + \operatorname{div}((1-\phi)\varrho_m \boldsymbol{v}_m) = -r_f ,$$

- Linear momenta balances (for individual components)

$$\frac{\partial(\phi\varrho_f \boldsymbol{v}_f)}{\partial t} + \operatorname{div}(\phi\varrho_f \boldsymbol{v}_f \otimes \boldsymbol{v}_f) = -\phi\nabla \mathbf{P}_f + \operatorname{div}(\phi \mathbf{S}_f) + \underbrace{r_f \boldsymbol{v}_S}_{\text{mass-mom. transfer}}$$
$$+ (\underbrace{\mathbf{P}_S}_{\text{surf. pressure}} - \mathbf{P}_f)\nabla\phi + \varrho_f\phi\mathbf{g} + \underbrace{\mathbf{h}_f}_{\text{gen. drag}} ,$$

$$\frac{\partial((1-\phi)\varrho_m \boldsymbol{v}_m)}{\partial t} + \operatorname{div}((1-\phi)\varrho_m \boldsymbol{v}_m \otimes \boldsymbol{v}_m) = -(1-\phi)\nabla \mathbf{P}_m + \operatorname{div}((1-\phi)\mathbf{S}_m) - r_f \boldsymbol{v}_S$$
$$- (\mathbf{P}_S - \mathbf{P}_m)\nabla\phi + \varrho_m(1-\phi)\mathbf{g} + \mathbf{h}_m ,$$

Binary mixture - balance laws

- Energy balance (for the mixture as a whole)

$$\frac{\partial}{\partial t}\left(\phi\varrho_{\rm f}(e_{\rm f}+\frac{1}{2}|\mathbf{v}_{\rm f}|^2)+(1-\phi)\varrho_{\rm m}(e_{\rm m}+\frac{1}{2}|\mathbf{v}_{\rm m}|^2)+\underbrace{\phi_S e_S}_{\text{s. energy d.}}\right)$$

$$+\quad \operatorname{div}\left(\phi\varrho_{\rm f}(e_{\rm f}+\frac{1}{2}|\mathbf{v}_{\rm f}|^2)\mathbf{v}_{\rm f}+(1-\phi)\varrho_{\rm m}(e_{\rm m}+\frac{1}{2}|\mathbf{v}_{\rm m}|^2)\mathbf{v}_{\rm m}+\phi_S e_S \mathbf{v}_S\right)$$

$$=\quad Q-\operatorname{div}\mathbf{q}+\operatorname{div}\left(-\phi\mathbf{P}_{\rm f}\mathbf{v}_{\rm f}-(1-\phi)P_{\rm m}\mathbf{v}_{\rm m}+\phi\mathbf{S}_{\rm f}\mathbf{v}_{\rm f}+(1-\phi)\mathbf{S}_{\rm m}\mathbf{v}_{\rm m}+\underbrace{\phi_S\sigma\mathbf{v}_S}_{\text{s. mech. power}}\right)$$

$$+\quad \phi\varrho_{\rm f}\mathbf{v}_{\rm f}\cdot\mathbf{g}+(1-\phi)\varrho_{\rm m}\mathbf{v}_{\rm m}\cdot\mathbf{g}\ ,$$

- Entropy balance (for the mixture as a whole)

$$\frac{\partial}{\partial t}\left(\phi\varrho_{\rm f}\eta_{\rm f}+(1-\phi)\varrho_{\rm m}\eta_{\rm m}+\underbrace{\phi_S\eta_S}_{\text{s. entropy d.}}\right)$$

$$+\quad \operatorname{div}\left(\phi\varrho_{\rm f}\eta_{\rm f}\mathbf{v}_{\rm f}+(1-\phi)\varrho_{\rm m}\eta_{\rm m}\mathbf{v}_{\rm m}+\phi_S e_S\mathbf{v}_S\right)=\operatorname{div}\mathbf{J}+\xi$$

Binary mixture - Constitutive theory - Incompressible case

- Existing theory: Bercovici, Ricard, Šrámek (Bercovici et al. (2001), Šrámek et al. (2007))
- Independent dissipation mechanisms (Šrámek et al., 2007)

$$
\begin{aligned}
\vartheta\zeta \quad = \quad & -\mathbf{q}\cdot\frac{\nabla\vartheta}{\vartheta} && \text{heat flow}\\
+ \quad & c(\phi)|\boldsymbol{v}_\mathrm{f}-\boldsymbol{v}_\mathrm{m}|^2 && \text{drag diss.}\\
+ \quad & \phi\mathbf{S}_\mathrm{f}:\mathbf{D}^d(\boldsymbol{v}_\mathrm{f})+(1-\phi)\mathbf{S}_\mathrm{m}:\mathbf{D}^d(\boldsymbol{v}_\mathrm{m}) && \text{viscous shear diss.}\\
- \quad & \left((\mathbf{P}_\mathrm{m}-\mathbf{P}_\mathrm{f})+\sigma\frac{d\phi_S}{d\phi}\right)((1-\omega)(1-\phi)\,\mathrm{div}\,\boldsymbol{v}_\mathrm{m}+\phi\omega\,\mathrm{div}\,\boldsymbol{v}_\mathrm{f}) && \text{compaction}\\
+ \quad & r_\mathrm{f}\left((\mu_\mathrm{m}-\mu_\mathrm{f})-\frac{\varrho_S}{\varrho_\mathrm{f}\varrho_\mathrm{m}}\left((\mathbf{P}_\mathrm{m}-\mathbf{P}_\mathrm{f})+\sigma\frac{d\phi_S}{d\phi}\right)+\frac{1-2\omega}{2}|\boldsymbol{v}_r|^2\right) && \text{melting}
\end{aligned}
$$

- Rate of entropy production in the form of product of thermodynamic fluxes and affinities

$$\xi = \mathbf{J}\cdot\mathbf{A}$$

- Linear relations proposed between $\mathbf{J}_\mathrm{i}$ and $\mathbf{A}_\mathrm{i}$ (i.e. no cross-effects considered)

Binary mixture - Constitutive theory - Incompressible case

- Generalized Clausius-Clapeyron relation

$$\underbrace{(\mu_m - \mu_f)}_{\text{chem. pot. diff.}} - \frac{\varrho_S}{\varrho_f \varrho_m} \underbrace{\left((\mathbf{P}_m - \mathbf{P}_f) + \sigma \frac{d\phi_S}{d\phi} \right)}_{\text{dyn. press. diff.}} + \frac{1-2\omega}{2} |\boldsymbol{v}_f - \boldsymbol{v}_m|^2 = 0 \ ,$$

- Stress relations (viscous fluid model)

$$\mathbf{S}_f = 2\nu_f \mathbf{D}^d(\boldsymbol{v}_f), \qquad \mathbf{S}_m = 2\nu_m \mathbf{D}^d(\boldsymbol{v}_m)$$

- Fourier law

$$\mathbf{q} = -\kappa(\phi)\nabla\vartheta$$

- Dynamic pressure-difference

$$\mathbf{P}_m - \mathbf{P}_f + \underbrace{\sigma \frac{d\phi_S}{d\phi}}_{\text{Laplace-Young}} = -\mu_0 \frac{\mu_f + \mu_m}{\phi(1-\phi)} \underbrace{\left((1-\omega)(1-\phi)\,\mathrm{div}\,\boldsymbol{v}_m - \phi\omega\,\mathrm{div}\,\boldsymbol{v}_f \right)}_{\text{compaction rate}}$$

Scaling → Model reduction → Stokes-"Darcy"-Fourier

- Balances of mass

$$\begin{aligned} \frac{\partial \phi}{\partial t} + \mathrm{div}\,(\phi \boldsymbol{v}_\mathrm{f}) &= \frac{r_\mathrm{f}}{\varrho_\mathrm{f}}, \\ \mathrm{div}\,((1-\phi)\boldsymbol{v}_\mathrm{m}) + \mathrm{div}\,(\phi \boldsymbol{v}_\mathrm{f}) &= r_\mathrm{f}\left(\frac{\varrho_\mathrm{m}-\varrho_\mathrm{f}}{\varrho_\mathrm{m}\varrho_\mathrm{f}}\right), \end{aligned}$$

- Linear momenta balances ($\Pi = \mathbf{P}_\mathrm{f} - \mathbf{P}_\mathrm{m}^\mathrm{ref}$)

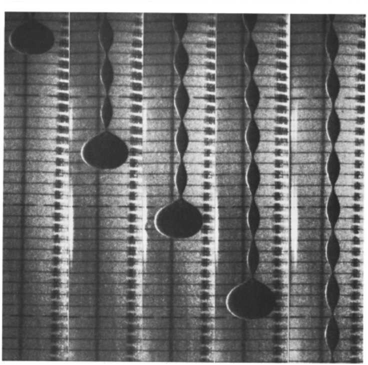
$$\begin{aligned} c(\phi)(\boldsymbol{v}_\mathrm{f}-\boldsymbol{v}_\mathrm{m}) &= -\phi\,(\nabla\Pi + (\varrho_\mathrm{m}-\varrho_\mathrm{f})\mathbf{g}) \\ \nabla\Pi &= -\phi(\varrho_\mathrm{m}-\varrho_\mathrm{f})\mathbf{g} + \underbrace{\nabla(\phi_S(\phi)\sigma)}_{\text{surface tension}} + \underbrace{\mathrm{div}\,(2(1-\phi)\nu_\mathrm{m}\mathbf{D}^d(\boldsymbol{v}_\mathrm{m}))}_{\text{matrix visc. def.}} \\ &+ \underbrace{\nabla\left((1-\phi)\left(\sigma\frac{d\phi_S(\phi)}{d\phi} - \frac{\mu_0\nu_\mathrm{m}}{\phi}\,\mathrm{div}\,\boldsymbol{v}_\mathrm{m}\right)\right)}_{\text{dyn. pressure difference}} \end{aligned}$$

- Energy balance

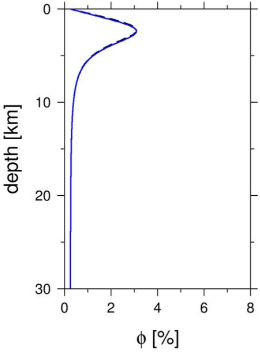
$$\begin{aligned} &\phi\rho_\mathrm{f} c_\mathrm{f}\left(\frac{\partial\vartheta}{\partial t} + \boldsymbol{v}_\mathrm{f}\cdot\nabla\vartheta\right) + (1-\phi)\rho_\mathrm{m} c_\mathrm{m}\left(\frac{\partial\vartheta}{\partial t} + \boldsymbol{v}_\mathrm{m}\cdot\nabla\vartheta\right) \\ - \quad &\vartheta\frac{\partial}{\partial t}\left(\phi_S(\phi)\frac{d\sigma}{d\vartheta}\right) + Lr_\mathrm{f} = Q + \mathrm{div}\left(\kappa(\phi)\nabla\vartheta\right) + c(\phi)|\boldsymbol{v}_\mathrm{r}|^2\ , \end{aligned}$$

Reduced model - qualitative behavior of solutions

- Strong mechanical matrix-fluid coupling due to viscous deformation of the matrix
- Wave-trains, solitary waves - experimentally observed



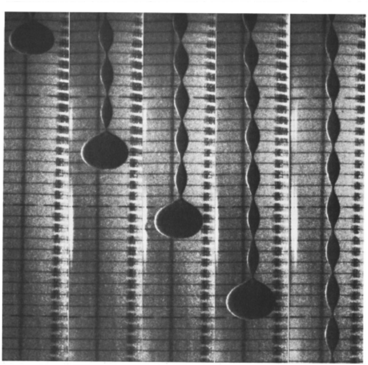
from Olson & Christensen (1986)



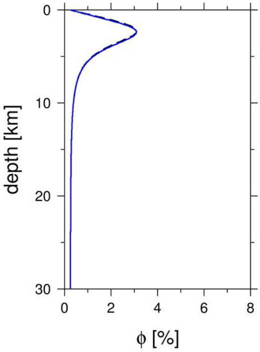
K. Kalousová

Reduced model - qualitative behavior of solutions

- Strong mechanical matrix-fluid coupling due to viscous deformation of the matrix
- Wave-trains, solitary waves - experimentally observed



from Olson & Christensen (1986)



K. Kalousová

Extensions: Viscous compressible case

- Assumption of incompressibility of the pure substances ($\varrho_{\rm f}=$ const., $\varrho_{\rm m}=$ const.) may be too restrictive (planetary interiors)
- Generalization of the Ricard - Bercovici - Šrámek model
- We assume compressible fluids with only shear viscosities

$$
\begin{aligned}
\mathbf{T}_{\rm f} &= -\mathbf{P}_{\rm f}(\varrho_{\rm f},\vartheta)\mathbf{I} + 2\nu_{\rm f}\mathbf{D}^d(\boldsymbol{v}_{\rm f}) \\
\mathbf{T}_{\rm m} &= -\mathbf{P}_{\rm m}(\varrho_{\rm m},\vartheta)\mathbf{I} + 2\nu_{\rm m}\mathbf{D}^d(\boldsymbol{v}_{\rm m})
\end{aligned}
$$

- Starting point - macroscopic equilibrium Gibbs relation:

$$
\begin{aligned}
\vartheta dS_{\rm f} &= dU_{\rm f} + \tilde{\mathbf{P}}_{\rm f} dV_{\rm f} - \tilde{\mu}_{\rm f} dm_{\rm f} \\
\vartheta dS_{\rm m} &= dU_{\rm m} + \tilde{\mathbf{P}}_{\rm m} dV_{\rm m} - \tilde{\mu}_{\rm m} dm_{\rm m}
\end{aligned}
$$

where equilibrium pressures $\tilde{\mathbf{P}}_{\rm f}$, $\tilde{\mathbf{P}}_{\rm m}$ are identified as:

$$
\begin{aligned}
\tilde{\mathbf{P}}_{\rm f} &= \left(\mathbf{P}_{\rm f} + \omega(\mathbf{P}_{\rm m} - \mathbf{P}_{\rm f} + \sigma\frac{d\phi_S}{d\phi})\right) \\
\tilde{\mathbf{P}}_{\rm m} &= \left(\mathbf{P}_{\rm m} - (1-\omega)(\mathbf{P}_{\rm m} - \mathbf{P}_{\rm f} + \sigma\frac{d\phi_S}{d\phi})\right)
\end{aligned}
$$

and associated chemical potentials:

$$
\tilde{\mu}_{\rm f} = e_{\rm f} - \vartheta\eta_{\rm f} + \frac{\tilde{\mathbf{P}}_{\rm f}}{\varrho_{\rm f}} \qquad \tilde{\mu}_{\rm m} = e_{\rm m} - \vartheta\eta_{\rm m} + \frac{\tilde{\mathbf{P}}_{\rm m}}{\varrho_{\rm m}}
$$

Extensions: Viscous compressible case - Scaling → Model reduction

- Balances of mass

$$\begin{aligned} \frac{\partial \phi}{\partial t} + \operatorname{div}(\phi \boldsymbol{v}_\mathrm{f}) &= \frac{r_\mathrm{f}}{\varrho_\mathrm{f}} - \frac{\phi}{\varrho_\mathrm{f}} \frac{D_\mathrm{f}\varrho_\mathrm{f}}{Dt}, \\ \operatorname{div}((1-\phi)\boldsymbol{v}_\mathrm{m}) + \operatorname{div}(\phi \boldsymbol{v}_\mathrm{f}) &= r_\mathrm{f}\left(\frac{\varrho_\mathrm{m} - \varrho_\mathrm{f}}{\varrho_\mathrm{m}\varrho_\mathrm{f}}\right) - \frac{\phi}{\varrho_\mathrm{f}} \frac{D_\mathrm{f}\varrho_\mathrm{f}}{Dt} - \frac{1-\phi}{\varrho_\mathrm{m}} \frac{D_\mathrm{m}\varrho_\mathrm{m}}{Dt}, \end{aligned}$$

- Linear momenta balances ($\Pi = \mathbf{P}_\mathrm{f} - \mathbf{P}_\mathrm{m}^\mathrm{ref}$)

$$\begin{aligned} c(\phi)\boldsymbol{v}_\mathrm{r} &= -\phi\left(\nabla\Pi + (\varrho_\mathrm{m} - \varrho_\mathrm{f})\mathbf{g}\right) \\ \nabla\Pi &= -\phi(\varrho_\mathrm{m} - \varrho_\mathrm{f})\mathbf{g} + \nabla(\phi_S(\phi)\sigma) + \operatorname{div}\left(2(1-\phi)\nu_\mathrm{m}\mathbf{D}^d(\boldsymbol{v}_\mathrm{m})\right) \\ &+ \nabla\left((1-\phi)\left(\sigma\frac{d\phi_S(\phi)}{d\phi} - \frac{\mu_0\nu_\mathrm{m}}{\phi}\operatorname{div}\boldsymbol{v}_\mathrm{m}\right)\right) \end{aligned}$$

- Energy balance

$$\begin{aligned} &\phi\varrho_\mathrm{f} c_\mathrm{f}\left(\frac{\partial\vartheta}{\partial t} + \boldsymbol{v}_\mathrm{f}\cdot\nabla\vartheta\right) + (1-\phi)\rho_\mathrm{m} c_\mathrm{m}\left(\frac{\partial\vartheta}{\partial t} + \boldsymbol{v}_\mathrm{m}\cdot\nabla\vartheta\right) \\ -\ &\vartheta\frac{\partial}{\partial t}\left(\phi_S(\phi)\frac{d\sigma}{d\vartheta}\right) + L r_\mathrm{f} = Q + \operatorname{div}\left(\kappa(\phi)\nabla\vartheta\right) + c(\phi)|\boldsymbol{v}_\mathrm{r}|^2, \end{aligned}$$

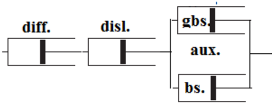
Extensions: Realistic ice viscosity

- Most two-phase models simplify the matrix and fluid rheologies by constant viscosities - very non-realistic approximation for ice
- Four deformational mechanisms: diffusion creep (diff), dislocation creep (disl), grain boundary sliding (gbs) and basal slip (bs), depending on: temperature, grain size d, pressure $\mathbf{P}$, and the second stress invariant σ_{II}

$$
\begin{aligned}
\mathbf{S}^\alpha &= 2\nu^\alpha \mathbf{D}^{d\,\alpha} \\
\nu^\alpha &= \frac{1}{2}\frac{d^{m^\alpha}}{A^\alpha \sigma_{II}^{n^\alpha - 1}} \exp\left(\frac{E^{\alpha *} + \mathbf{P} V^{\alpha *}}{R\vartheta}\right),
\end{aligned}
$$

- Combined rheology (IMPLICIT $\mathbf{S}$-$\mathbf{D}^d$ relation):

$$
\begin{aligned}
\mathbf{D}^d &= \mathbf{D}^{d\,\mathrm{diff}} + \mathbf{D}^{d\,\mathrm{disl}} + \mathbf{D}^{d\,\mathrm{aux}} \\
\mathbf{S} &= (A^{\mathrm{bs}})^{-\frac{1}{n^{\mathrm{bs}}}} d^{\frac{m^{\mathrm{bs}}}{n^{\mathrm{bs}}}} (\mathbf{D}^{d\,\mathrm{aux}}_{II})^{\frac{1-n^{\mathrm{bs}}}{n^{\mathrm{bs}}}} \mathbf{D}^{d\,\mathrm{aux}} \\
&+ (A^{\mathrm{gbs}})^{-\frac{1}{n^{\mathrm{gbs}}}} d^{\frac{m^{\mathrm{gbs}}}{n^{\mathrm{gbs}}}} (\mathbf{D}^{d\,\mathrm{aux}}_{II})^{\frac{1-n^{\mathrm{gbs}}}{n^{\mathrm{gbs}}}} \mathbf{D}^{d\,\mathrm{aux}}
\end{aligned}
$$



Extensions: Realistic ice viscosity

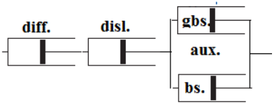
- Most two-phase models simplify the matrix and fluid rheologies by constant viscosities - very non-realistic approximation for ice
- Four deformational mechanisms: diffusion creep (diff), dislocation creep (disl), grain boundary sliding (gbs) and basal slip (bs), depending on: temperature, grain size d, pressure $\mathbf{P}$, and the second stress invariant σ_{II}

$$\mathbf{S}^{\alpha} = 2\nu^{\alpha}\mathbf{D}^{d\,\alpha}$$

$$\nu^{\alpha} = \frac{1}{2}\frac{d^{m^{\alpha}}}{A^{\alpha}\sigma_{II}^{n^{\alpha}-1}} \exp\left(\frac{E^{\alpha*} + \mathbf{P}V^{\alpha*}}{R\vartheta}\right),$$

- Often used approximation - explicit, but non-invertible $\mathbf{D}(\mathbf{S})$ relation:

$$\frac{1}{\nu} = \frac{1}{\nu^{\mathrm{diff}}} + \frac{1}{\nu^{\mathrm{disl}}} + \frac{1}{\nu^{\mathrm{gbs}} + \nu^{\mathrm{bs}}}.$$



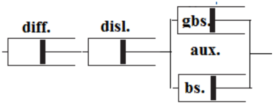
Extensions: Realistic ice viscosity

- Most two-phase models simplify the matrix and fluid rheologies by constant viscosities - very non-realistic approximation for ice
- Four deformational mechanisms: diffusion creep (diff), dislocation creep (disl), grain boundary sliding (gbs) and basal slip (bs), depending on: temperature, grain size d, pressure $\mathbf{P}$, and the second stress invariant σ_{II}

$$\begin{aligned} \mathbf{S}^\alpha &= 2\nu^\alpha \mathbf{D}^{d\,\alpha} \\ \nu^\alpha &= \frac{1}{2}\frac{d^{m^\alpha}}{A^\alpha \sigma_{II}^{n^\alpha - 1}} \exp\left(\frac{E^{\alpha *} + \mathbf{P} V^{\alpha *}}{R\vartheta}\right), \end{aligned}$$

- Often used approximation - explicit, but non-invertible $\mathbf{D}(\mathbf{S})$ relation:

$$\frac{1}{\nu} = \frac{1}{\nu^{\mathrm{diff}}} + \frac{1}{\nu^{\mathrm{disl}}} + \frac{1}{\nu^{\mathrm{gbs}} + \nu^{\mathrm{bs}}}.$$



- Porosity weakening (lubrication)

$$\nu(\phi) \doteq \nu^{\mathrm{pure}} \exp(-45\phi)$$

Extensions: Realistic ice viscosity

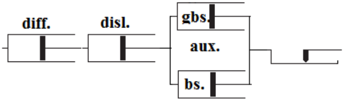
- Most two-phase models simplify the matrix and fluid rheologies by constant viscosities - very non-realistic approximation for ice
- Four deformational mechanisms: diffusion creep (diff), dislocation creep (disl), grain boundary sliding (gbs) and basal slip (bs), depending on: temperature, grain size d, pressure $\mathbf{P}$, and the second stress invariant σ_{II}

$$\mathbf{S}^\alpha = 2\nu^\alpha \mathbf{D}^{d\,\alpha}$$

$$\nu^\alpha = \frac{1}{2}\frac{d^{m^\alpha}}{A^\alpha \sigma_{II}^{n^\alpha - 1}} \exp\left(\frac{E^{\alpha *} + \mathbf{P} V^{\alpha *}}{R\vartheta}\right),$$

- Often used approximation - explicit, but non-invertible $\mathbf{D}(\mathbf{S})$ relation:

$$\frac{1}{\nu} = \frac{1}{\nu^{\mathrm{diff}}} + \frac{1}{\nu^{\mathrm{disl}}} + \frac{1}{\nu^{\mathrm{gbs}} + \nu^{\mathrm{bs}}}.$$



- Porosity weakening (lubrication)

$$\nu(\phi) \doteq \nu^{\mathrm{pure}} \exp(-45\phi)$$

- Plastic-like stress-limiter

$$\frac{1}{\tilde{\nu}} = \frac{1}{\nu} + \frac{2\|\mathbf{D}\|}{\sigma_{Yield}}$$

Numerical implementation

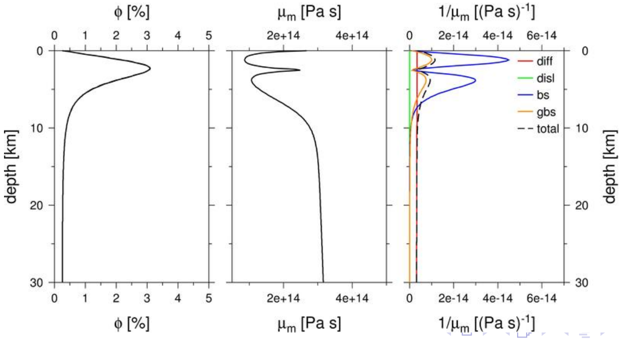
K. Kalousová (Ph.D.)

- Fortran90 (1d)
  - Allows zero compaction length regime - shocks
  - space: FV+ENO & FEM
  - time: RK schemes
  - tests: shock velocity (Rankine-Hugoniot condition), wavetrain propagation (*Spiegelman*, 1993), phase velocity (*Rabinowicz et al.*, 2002)

- FEniCS (http://fenicsproject.org) (1d, 2d)
  - space: FEM (CG Taylor-Hood), SUPG stabilization
  - time: Crank-Nicolson (semi-implicit, 2^{nd} order) + predictor-corrector Stokes-Darcy – Heat eq.
  - tests: comparison with 1d Fortran90, convection benchmark (*Blankenbach et al.*, 1989)

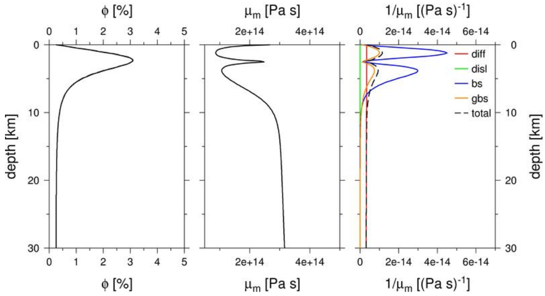
Numerical experiments - Sensitivity study (1d)

- Parametric study (spatial 1D) of rheology-related effects never studied in the given two-phase flow context (S. et al., 2014)
- Effects of ice deformation mechanisms, temperature, porosity-weakening effects
- Possibility by a parametrization by constant ice viscosity?
- Moderate or small effects on global scale (effective permeability of the whole ice layer)
- Possibly very large effects at local scale
- Example (composite rheology effects):



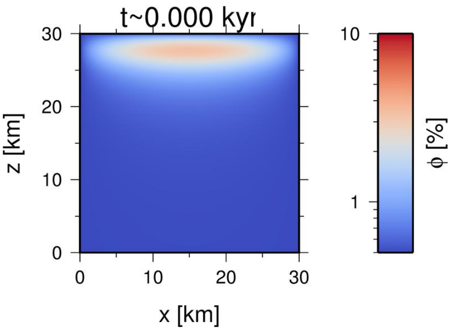
Numerical experiments - Sensitivity study (1d)

- Parametric study (spatial 1D) of rheology-related effects never studied in the given two-phase flow context (S. et al., 2014)
- Effects of ice deformation mechanisms, temperature, porosity-weakening effects
- Possibility by a parametrization by constant ice viscosity?
- Moderate or small effects on global scale (effective permeability of the whole ice layer)
- Possibly very large effects at local scale
- Example (composite rheology effects):

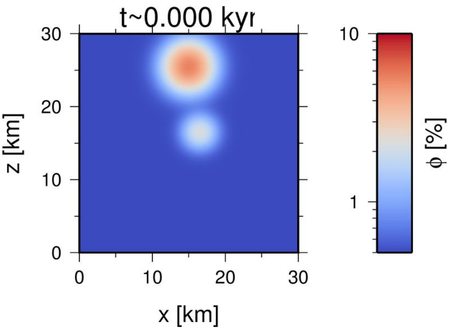


Numerical experiments: 2D temperate case

- ϱ_f, ϱ_m constant
- ν_f, ν_m constant
- Flow localization, channeling



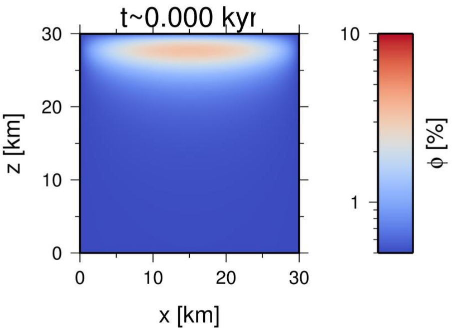
K. Kalousová



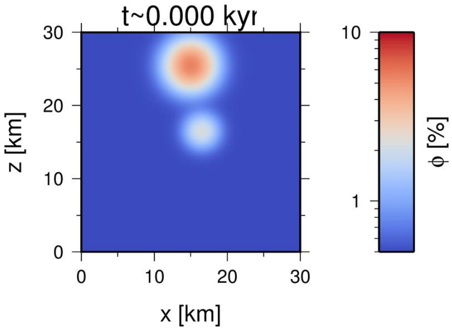
K. Kalousová

Numerical experiments: 2D temperate case

- ϱ_{f}, ϱ_{m} constant
- ν_{f}, ν_{m} constant
- Flow localization, channeling



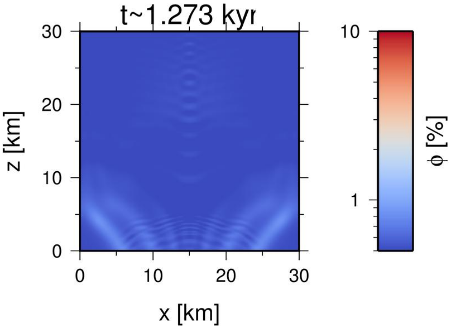
K. Kalousová

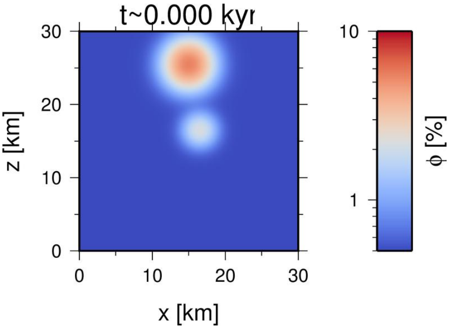


K. Kalousová

Numerical experiments: 2D temperate case

- $\varrho_{\rm f}$, $\varrho_{\rm m}$ constant
- $\nu_{\rm f}$, $\nu_{\rm m}$ constant
- Flow localization, channeling





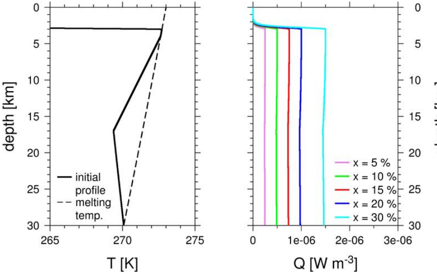
K. Kalousová

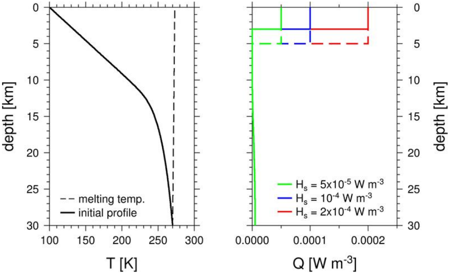
Europa: Ice melting and water transport in the ice shell (1d)

Kalousová et al., 2014:

- hot plume model
  - tidal heating (*Tobie et al.*, 2003):
    $H_\mathrm{t} = \frac{2H_\mathrm{t}^\mathrm{max}}{\mu_\mathrm{m}/\mu_\mathrm{m}^\mathrm{max} + \mu_\mathrm{m}^\mathrm{max}/\mu_\mathrm{m}}$
  - convective cooling:
    $Q_\mathrm{t} = H_\mathrm{t} - H_\mathrm{cool} = xH_\mathrm{t}$

- strike-slip fault model
  - tidal heating, no convection ($x=1$):
    $Q_\mathrm{t} = H_\mathrm{t} = \frac{2H_\mathrm{t}^\mathrm{max}}{\mu_\mathrm{m}/\mu_\mathrm{m}^\mathrm{max} + \mu_\mathrm{m}^\mathrm{max}/\mu_\mathrm{m}}$
  - shear heating:
    $Q_\mathrm{s}(z) = H_\mathrm{s} \exp(-\gamma_\mathrm{s}\phi) \qquad z \geq z_\mathrm{s}$

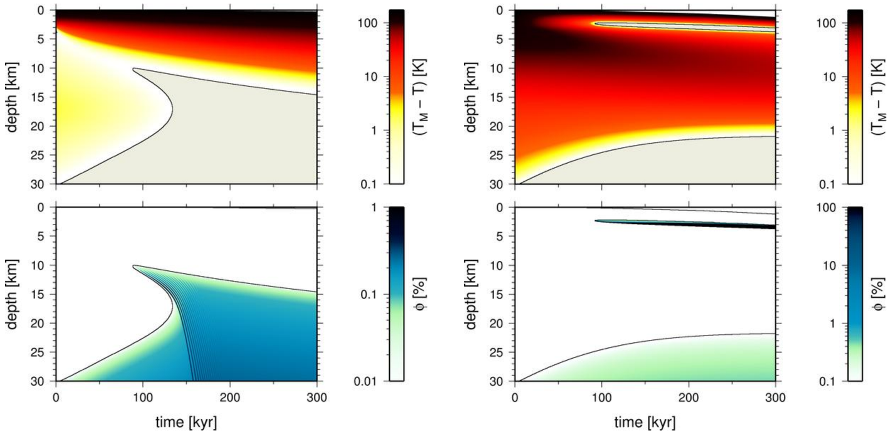




Europa: Ice melting and water transport in the ice shell (1d)

Kalousová et al., 2014:

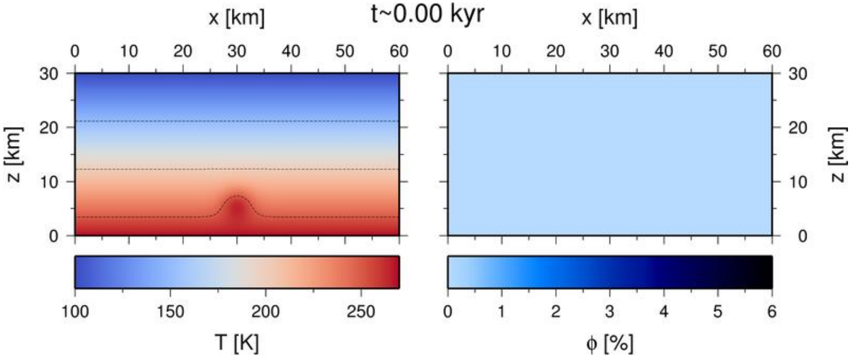
- Hot plume model
- Strike-slip fault model



- accumulation of liquid water not possible within hot plumes
- accumulation of liquid water possible at strike-slip faults

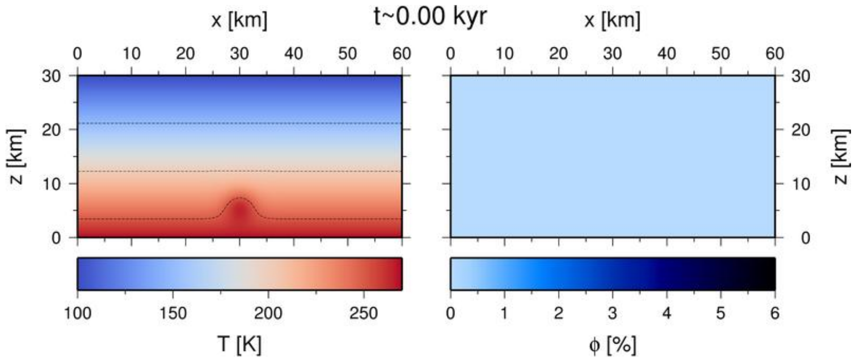
Europa: Ice melting and water transport in the ice shell (2d)

- Impermeable case ($\mathbf{v}_\mathrm{f}$=$\mathbf{v}_\mathrm{m}$), Thermal convection + melting + compaction
- Hot plume model
  $H_\mathrm{t}^\mathrm{max}$=$3\times10^{-6}$ W m^{-3}, d=0.7 mm



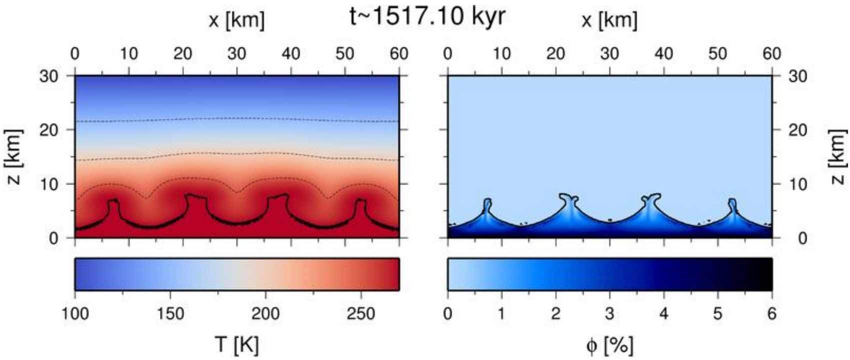
Europa: Ice melting and water transport in the ice shell (2d)

- Impermeable case ($\mathbf{v}_\mathrm{f}=\mathbf{v}_\mathrm{m}$), Thermal convection + melting + compaction
- Hot plume model
  $H_\mathrm{t}^\mathrm{max}=3\times10^{-6}$ W m^{-3}, $d=0.7$ mm



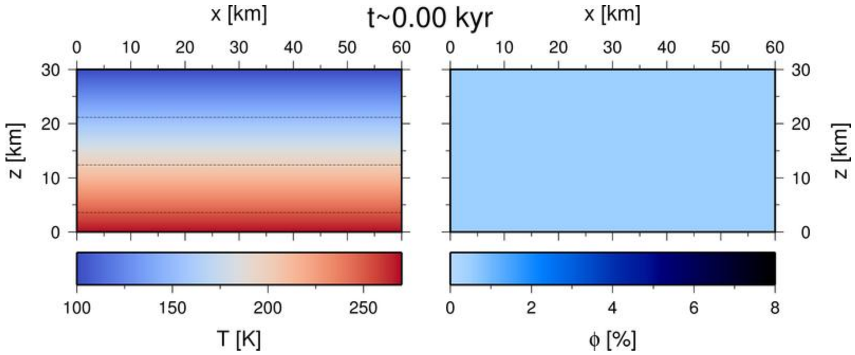
Europa: Ice melting and water transport in the ice shell (2d)

- Impermeable case ($\mathbf{v}_\mathrm{f}=\mathbf{v}_\mathrm{m}$), Thermal convection + melting + compaction
- Hot plume model
  $H_\mathrm{t}^\mathrm{max}=3\times10^{-6}$ W m^{-3}, $d=0.7$ mm



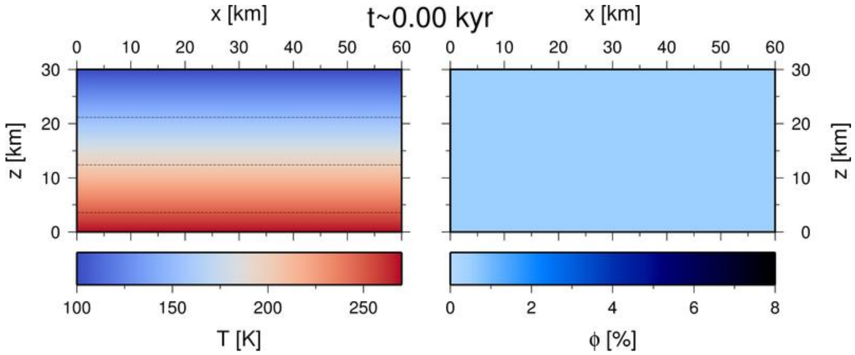
Ice melting and water transport in Europa's ice shell (2d)

- Impermeable case ($\mathbf{v}_\mathrm{f}=\mathbf{v}_\mathrm{m}$), Thermal convection + melting + compaction
- Strike-slip fault model
  $H^{\max}=5\times10^{-6}$ W m^{-3}, $H^{\mathrm{s}}=2\times10^{-4}$ W m^{-3}, d=0.7 mm, $\gamma_\mathrm{m}=\gamma_\mathrm{s}=45$



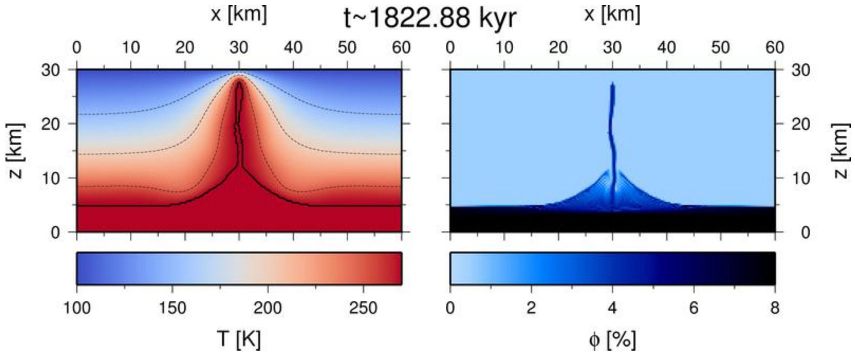
Ice melting and water transport in Europa's ice shell (2d)

- Impermeable case ($\mathbf{v}_\mathrm{f}=\mathbf{v}_\mathrm{m}$), Thermal convection + melting + compaction
- Strike-slip fault model
  $H^{\max}=5\times10^{-6}$ W m^{-3}, $H^{\mathrm{s}}=2\times10^{-4}$ W m^{-3}, $d=0.7$ mm, $\gamma_\mathrm{m}=\gamma_\mathrm{s}=45$



Ice melting and water transport in Europa's ice shell (2d)

- Impermeable case ($\mathbf{v}_\mathrm{f}=\mathbf{v}_\mathrm{m}$), Thermal convection + melting + compaction
- Strike-slip fault model
  $H^{\max}=5\times10^{-6}$ W m^{-3}, $H^{\mathrm{s}}=2\times10^{-4}$ W m^{-3}, $d=0.7$ mm, $\gamma_\mathrm{m}=\gamma_\mathrm{s}=45$



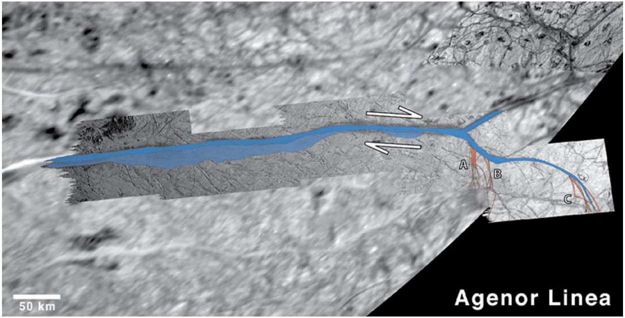
Europa: Summary

- Hot plumes:
  - Melting possible for $H_{\mathrm{t}}^{\mathrm{max}} \gtrsim 3\times10^{-6}$ W m^{-3} and $d\sim$0.5–1 mm
  - Meltwater quickly ($\lesssim$ few 100 of kyr) transported downwards
  - $\rightarrow$ Accumulation of liquid water at shallow depths unlikely
- Strike-slip faults:
  - Melting possible $\sim$3 km below surface for $H_{\mathrm{s}} \gtrsim 2\times10^{-4}$ W m^{-3}
  - Reservoir of $\phi\sim$10% stable for at least 1000 kyr
  - $\rightarrow$ Liquid water below strike-slip faults possibly stable for several 100 of kyr if the ice below is free of fractures & sufficiently cold

Search for liquid water at Europa

What would be the best candidates to search for liquid water on Europa with a radar instrument?

- **recently active strike-slip faults**
- late stage of fracturing + reactivation of many lineaments as strike-slip faults: Agenor Linea is a good candidate for recent or even current activity (*Prockter et al.*, 2000; *Hoyer et al.*, 2014)



Perspectives

- Further model development
  - Liquid water transport by micropores through temperate parts of the shell → two-phase thermal convection
  - Brittle rheology (visco-plastic)
  - Grain-size evolution
  - Free surface evolution
  - Salinity evolution and effect of salt on T_{M} and buoyancy → two-phase thermo-chemical convection
  - Study of water transport by hydrofracturing + its implementation if significant
  - Improvement of the tidal heating models - (3d) viscoelastic model diurnal response model for plume/strike slip domain
- Possible applications
  - Enceladus - possibility to form regional ocean; shallow melting potentially connected with erupting jets
  - Ganymede, Titan - adaptation of developed formalism for deep layers of HP ices → chemical transport between rocky interior and internal ocean

Thank you for your attention!