Modelling, simulation and benchmarking of fluid-structure interactions with contact

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Modelling, PDE analysis and computational mathematics in materials science
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Joint work with

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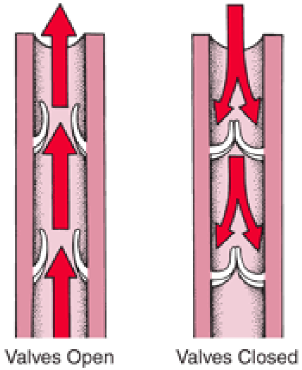
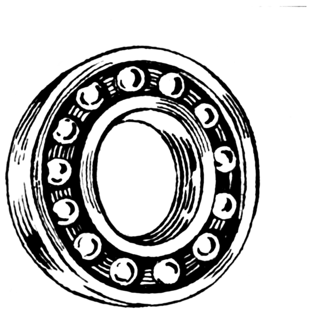
Motivation

Typically fluid or gas between contacting structures before/after contact

Vessel valves



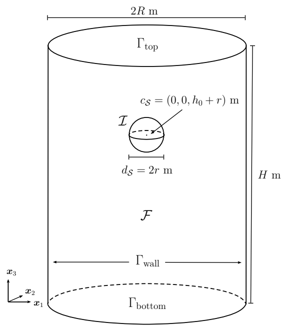
Ball bearing (Lubricant)

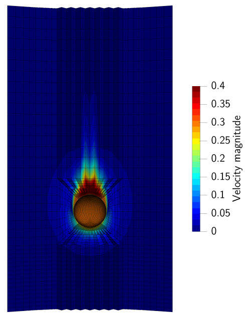


Experimental benchmark: Falling and bouncing elastic balls (PTFE, rubber) in a water-glycerine mixture

(Hagemeier, Thevenin, Richter, Int J Multiphase Flow (2020)

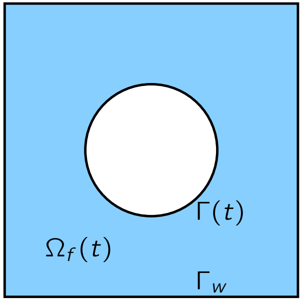
von Wahl, Richter, F., Hagemeier, Phys Fluids (2021))

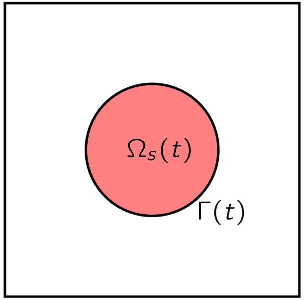




Overview

Fluid-structure interaction





Fluid problem
Incompr. Navier-Stokes equations in $\Omega_f(t)$

$$\rho_f(\partial_t u + u \cdot \nabla u) - \operatorname{div} \sigma_f(u, p) = f_f,$$
$$\operatorname{div} u = 0$$

Boundary conditions (no-slip)

$$u = 0 \quad \text{on } \Gamma_w \quad + \text{other bc}$$

Solid problem
Linear elastic material in $\Omega_s(t)$

$$\rho_s(\partial_t \dot{d} + \dot{d} \cdot \nabla d) - \operatorname{div} \sigma_s(d) = f_s,$$
$$\partial_t d + \dot{d} \cdot \nabla d = \dot{d}$$

Coupling conditions (no-slip)

$$u = \dot{d}, \quad \sigma_f n = \sigma_s n \quad \text{on } \Gamma(t).$$

No-collision paradox: The Navier-Stokes equations with no-slip boundary or interface conditions do not allow for contact (Hillairet/Hesla 2d; Gerart-Varet, Hillairet and Wang 3d)

Relaxed contact formulation

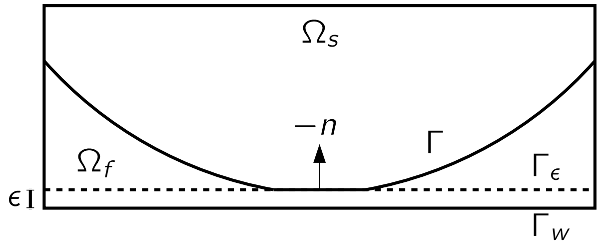
We impose the contact condition w.r.t to Γ_ϵ, such that a small fluid layer of size $\epsilon(h)$ remains

- The "**no-penetration**" condition reads

$$d_n \leq g_\epsilon := g_0 - \epsilon(h).$$

- The contact conditions are formulated with a Lagrange multiplier λ

$$d_n \leq g_\epsilon, \quad \lambda \leq 0, \quad (d_n - g_\epsilon)\lambda = 0$$



- Elimination of the Lagrange multiplier $(\lambda = \underbrace{\sigma_{s,nn} - \sigma_{f,nn}}_{[\![\sigma_{nn}]\!]})$

$$d_n \leq g_\epsilon, \quad [\![\sigma_{nn}]\!] \leq 0, \quad (d_n - g_\epsilon)[\![\sigma_{nn}]\!] = 0 \qquad (1)$$

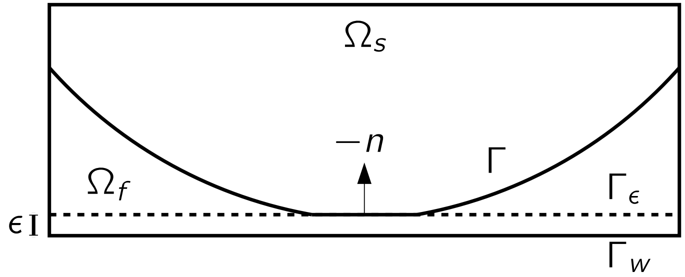
For arbitrary $\gamma > 0$ (1) is equivalent to (ALART & CURNIER '91)

$$[\![\sigma_{nn}]\!] = -\gamma \max\left\{d_n - g_\epsilon - \frac{1}{\gamma}[\![\sigma_{nn}]\!], 0\right\} =: -\gamma[P_\gamma([\![\sigma_{nn}]\!], d_n)]_+ \quad \text{on } \Gamma(t).$$

Problems

Relaxed contact formulation

$$d_n \le g_\epsilon, \quad [\![\sigma_{nn}]\!] \le 0, \quad (d_n - g_\epsilon)[\![\sigma_{nn}]\!] = 0$$



- The fluid forces $\sigma_{f,nn}$ in the fluid layer enter the contact dynamics
- As in particular the pressure p might take large values, these can have a non-physical impact on the contact dynamics
- We need a **physically motivated model** in the layer!

Idea:

- The layer appears due to surface roughness on a micro scale of thickness ϵ_p. We formulate a Darcy law in the layer and take $\epsilon_p \to 0$
- Similar ideas with a porous domain of thickness $\epsilon_p > 0$: Complex Biot-type equations (Ager, Schott, Vuong, Popp & Wall, 2019), Navier-Stokes-Brinkmann (Gerosa & Marsden, 2024)

Navier-Stokes-Darcy coupling

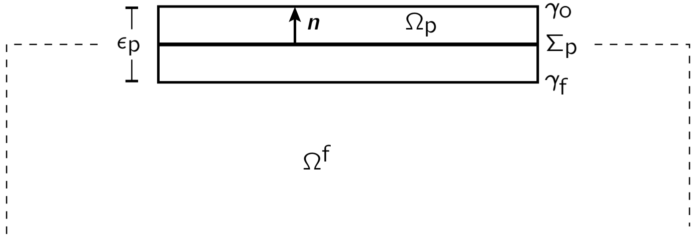
- In the porous domain Ω_p, we assume a **Darcy law**

$$\begin{cases} u_l + \boldsymbol{K}\nabla p_l = 0 & \text{in} \quad \Omega_p, \\ \nabla \cdot u_l = 0 & \text{in} \quad \Omega_p, \end{cases}$$

where $\boldsymbol{K} \in \mathbb{R}^{d\times d}$ with $\boldsymbol{K}\nabla p_l = K_\tau \nabla_\tau p_l + K_n \boldsymbol{\partial}_n p_l$.

- These equations are coupled to Navier-Stokes by means of the **Beavers-Joseph-Saffman** conditions

$$\begin{cases} \sigma_{f,n} = -p_l & \text{on} \quad \gamma_f, \\ u_n = u_l \cdot \boldsymbol{n} & \text{on} \quad \gamma_f, \\ \sigma_{f,n\tau} = -\dfrac{\alpha}{\sqrt{K_\tau \epsilon_p}} u_\tau & \text{on} \quad \gamma_f \end{cases}$$



- For $\epsilon_p \to 0$, we obtain on the mid-surface Σ_p (Martin et. al, 2005)

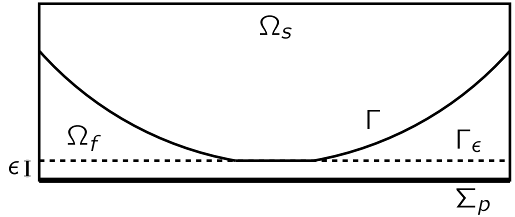
$$-\nabla_\tau \cdot (\epsilon_p K_\tau \nabla_\tau P_l) = u_n \quad \text{on} \quad \Sigma_p,$$

$$\sigma_{f,nn} = -P_l - \frac{\epsilon_p K_n^{-1}}{4} u_n \quad \text{on} \quad \Sigma_p,$$

$$\sigma_{f,n\tau} = -\frac{\alpha}{\sqrt{K_\tau \epsilon_p}} u_\tau \quad \text{on} \quad \Sigma_p$$

where $P_l := \frac{1}{2}\left(p_l|\gamma_f + p_l|\gamma_o\right)$.

Complete model



- **Solid problem** in $\Omega_s(t)$:

$$\begin{cases} \rho_s(\partial_t \dot{d} + \dot{d}\cdot\nabla\dot{d}) - \operatorname{div}\boldsymbol{\sigma}_s(d) = \rho_s \boldsymbol{f}_s, \\ \partial_t d + \dot{d}\cdot\nabla d = \dot{d}, \\ d = 0 \ \text{ on } \Gamma_s^d, \end{cases}$$

- **Fluid problem** in $\Omega_f(t)$

$$\begin{cases} \rho_f\big(\partial_t u + u\cdot\nabla u\big) - \operatorname{div}\boldsymbol{\sigma}_f(u,p) = \rho_f \boldsymbol{f}_f, \\ \operatorname{div} u = 0, \\ u = 0 \ \text{ on } \Gamma_f^d \end{cases}$$

- **Porous layer** (seepage)

$$\begin{cases} -\nabla_\tau\cdot(\epsilon_p K_\tau \nabla_\tau P_l) = u_n \quad \text{on} \quad \Sigma_p, \\ \epsilon_p K_\tau \tau\cdot\nabla_\tau P_l = 0 \quad \text{on} \quad \partial\Sigma_p, \end{cases}$$

- **Fluid-porous coupling** conditions:

$$\begin{cases} \sigma_{f,nn} = \underbrace{-P_l - \frac{\epsilon_p K_n^{-1}}{4} u_n}_{=:\,\sigma_p} \quad \text{on} \quad \Sigma_p, \\ \sigma_{f,n\tau} = -\frac{\alpha}{\sqrt{K_\tau \epsilon_p}} u_\tau \quad \text{on} \quad \Sigma_p. \end{cases}$$

- We combine **FSI** and **contact conditions** on the joint interface-contact surface $\Gamma(t)$ to

$$u = \dot{d}, \quad [\![\sigma_{nn}]\!] = -\gamma[P_\gamma([\![\sigma_{nn}]\!], d_n)]_+$$

Variational formulation (Fully Eulerian)

Simultaneous imposition of FSI interface and contact conditions on a **joint interface-contact surface** $\Gamma(t)$ by means of Nitsche's method (Burman, Fernández, F., Gerosa, CMAME 2022)

Find $u(t) \in \mathcal{V}_h$, $p(t) \in \mathcal{Q}_h$, $d(t), \dot{d} \in \mathcal{W}_h$, $P_l \in \mathcal{S}_h$, *such that*

$$
\begin{aligned}
a_f(u, p; v, q) + a_s(d, \dot{d}; w, z) - (\sigma_f(u, p)\boldsymbol{n}, w - v)_{\Gamma(t)} - \big(\dot{d} - u, \sigma_f(v, -q)\boldsymbol{n}\big)_{\Gamma(t)} \\
+ \frac{\gamma_{\text{fsi}}}{h}\big(\dot{d} - u, w - v\big)_{\Gamma(t)} + \frac{\gamma_c}{h}\big([P_\gamma([\![\sigma_{nn}]\!], d_n)]_+, w_n\big)_{\Gamma(t)} \\
- (\sigma_p, v_n)_{\Sigma_p} + (\epsilon_p K_\tau \nabla_\tau P_l, \nabla_\tau q_l)_{\Sigma_p} - \big(u_n, q_l\big)_{\Sigma_p} + \frac{\alpha}{\sqrt{K_\tau \epsilon_p}}(u_\tau, v_\tau)_{\Sigma_p} \\
= (f_f, v)_{\Omega_f(t)} + (f_s, w)_{\Omega_s(t)} \quad \forall v, q, w, z, q_l \in \mathcal{V}_h \times \mathcal{Q}_h \times \mathcal{W}_h \times \mathcal{W}_h \times \mathcal{S}_h
\end{aligned}
$$

where

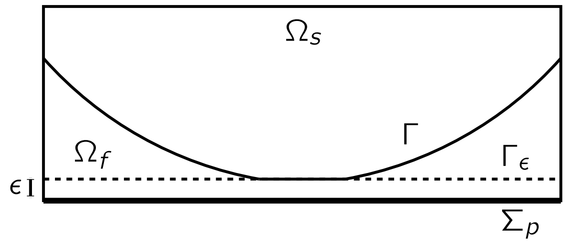
$$
a_f(u, p; v, q) := \rho_f\big(\partial_t u + u \cdot \nabla u, v\big)_{\Omega_f(t)} + (\sigma_f(u, p), \nabla v)_{\Omega_f(t)} + (\operatorname{div} u, q)_{\Omega_f(t)}
$$

$$
a_s(d, \dot{d}; w, z) := \big(\rho_s(\partial_t \dot{d} + \dot{d} \cdot \nabla \dot{d}), w\big)_{\Omega_s(t)} + (\sigma_s(d), \nabla w)_{\Omega_s(t)} + \big(\partial_t d + \dot{d} \cdot \nabla d - \dot{d}, z\big)_{\Omega_s(t)}
$$

Advantages & mechanical consistency

The **relaxed contact formulation** greatly simplifies the implementation:

- The coupling is for all times structure-fluid and fluid-porous medium (never structure-porous medium)
- Topology changes are avoided



Collisions are possible, even when no-slip is used on the FSI interface
(Champion, Fernández, Grandmont, Vergnet, Vidrascu 2024)

Mechanical consistency:

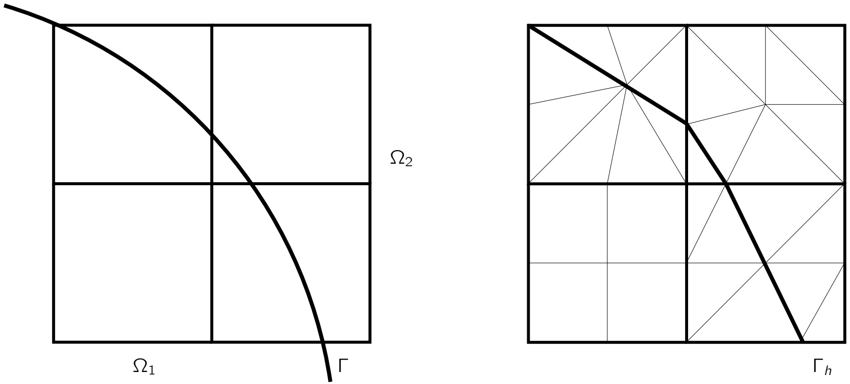
- If a part of $\Gamma(t)$ is in 'contact' with Σ_p (in the relaxed sense), it holds

$$\sigma_{\mathrm{f},nn}|_\Gamma \approx \sigma_\mathrm{p}|_{\Sigma_\mathrm{p}}.$$

- This gives a physical meaning to the fluid forces $\sigma_{\mathrm{f},nn}$ in the layer.

Discretisation: Fitted (locally modified) finite elements

- Fixed regular patch mesh independent of the interface location
- Split interface patches into eight triangles to resolve the interface
- Combination of P_1 and Q_1 finite elements (F. & Richter, SINUM 2014)



- Equal-order locally mod. FE with anisotropic edge-oriented pressure stabilisation (F., IJNMF, 2019)
- Time discretisation: Modified dG(0) scheme (F., Richter, M2AN, 2017)

Numerical example

- Fall of an elastic PTFE ball within a water-glycerin mixture (2d)
- Porous medium

$$\epsilon_p = 10^{-4},\ K_\tau = K_n = 10^{-2}$$

- Contact parameters

$$\gamma_c = 30\lambda_s,\ \epsilon_g = \frac{h_{\min}}{4}$$

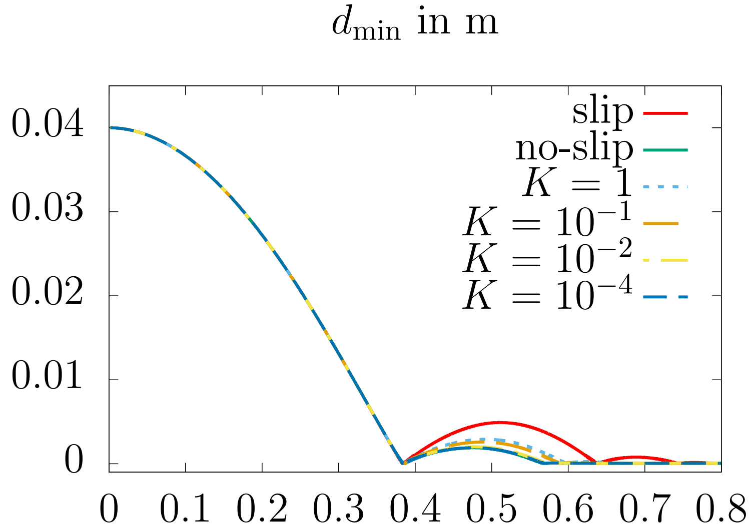
- Adaptive time-stepping

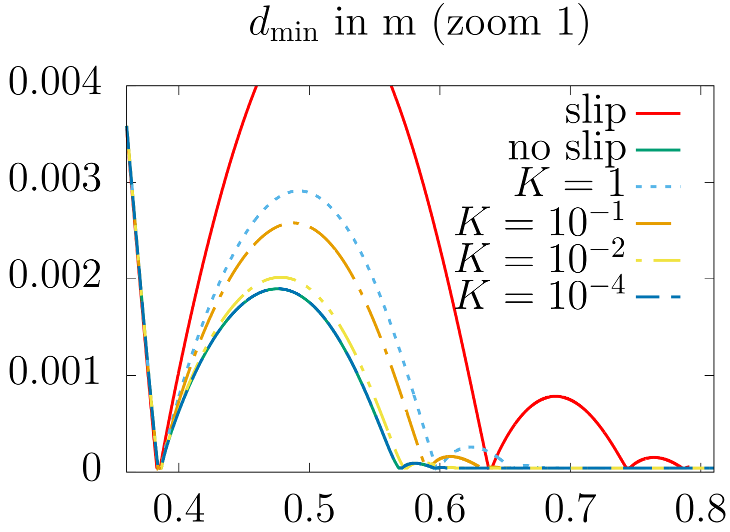
$$\delta t \in \left[\frac{1}{16\,000}, \frac{1}{500}\right] \text{s}$$

(movie)

Variation of the porous conductivity K

Minimal distance d_{min} to the ground over time for different conductivities K compared to a relaxation approach (without porous layer) with slip resp. no-slip conditions

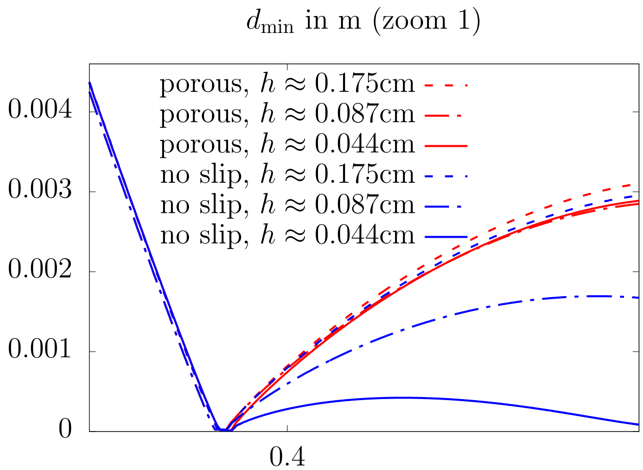


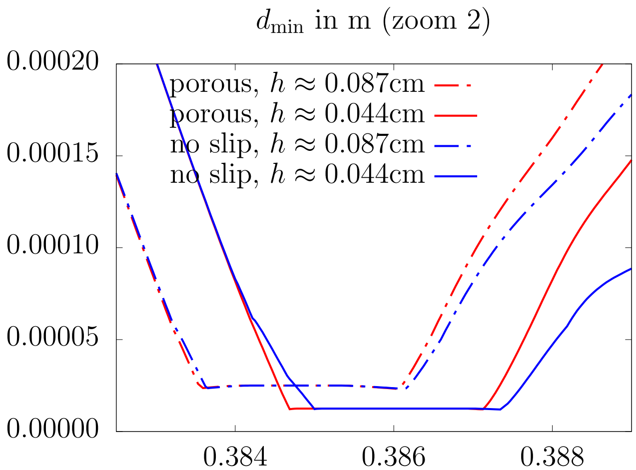


Results with porous layer lie between slip (larger bounce) and no-slip conditions

Comparison to relaxation without porous layer

Minimal distance $d_{\min}$ and zoom-in for different h





Solid velocities $\dot{d}_y$ at impact (t_i) and release (t_r)

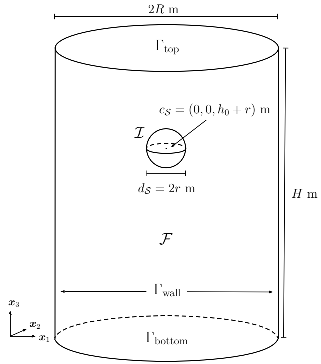
	With porous layer		Only no-slip	
h	$-\dot{d}_y(t_i)$	$\dot{d}_y(t_r)$	$-\dot{d}_y(t_i)$	$\dot{d}_y(t_r)$
$1.75 \cdot 10^{-3}$	$1.12 \cdot 10^{-1}$	$8.87 \cdot 10^{-2}$	$1.11 \cdot 10^{-1}$	$8.81 \cdot 10^{-2}$
$8.77 \cdot 10^{-2}$	$1.05 \cdot 10^{-1}$	$8.48 \cdot 10^{-2}$	$9.74 \cdot 10^{-2}$	$7.59 \cdot 10^{-2}$
$4.39 \cdot 10^{-2}$	$1.03 \cdot 10^{-1}$	$8.91 \cdot 10^{-2}$	$7.25 \cdot 10^{-2}$	$5.98 \cdot 10^{-2}$

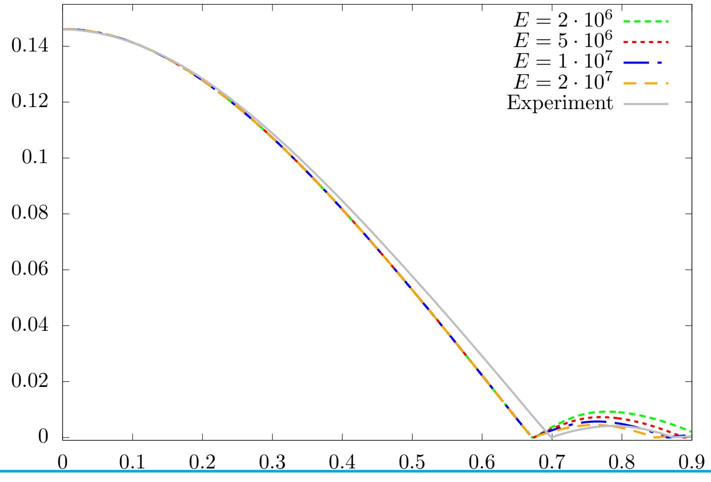
Comparison with experimental benchmark (2.5d)

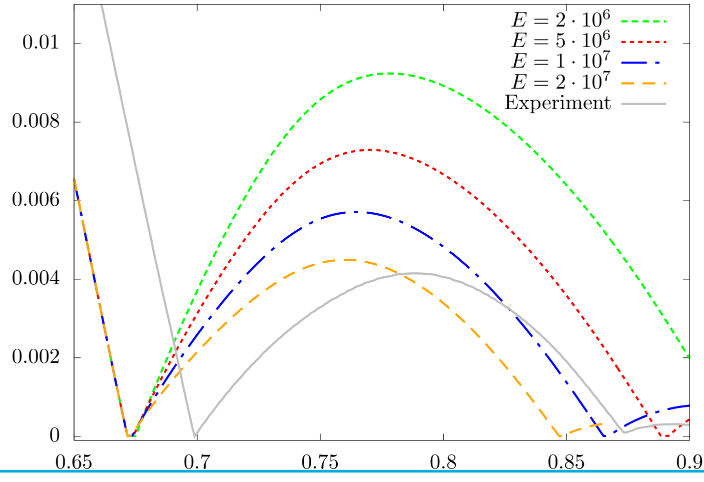
Falling rubber ball within water-glycerine mixture

(HAGEMEIER, THEVENIN, RICHTER, Int J Multiphase Flow (2020),
VON WAHL, RICHTER, F., HAGEMEIER, Phys Fluids (2021))

- Solid parameters $\nu = 0.4999$, $E \in [1.7 \cdot 10^6, 2.1 \cdot 10^7]$
- Minimal distance d_{min} and zoom-in for different Young's moduli E:



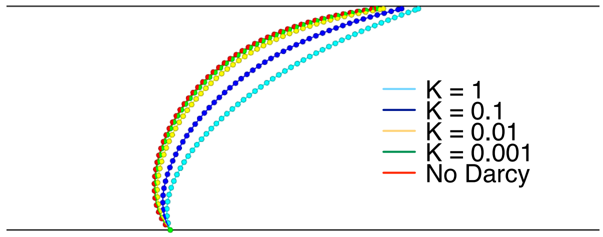




Conclusion and outlook

Conclusion

- Mechanically consistent and easy implementable model for FSI and contact considering seepage
- Applied also to a thin elastic solid (beam model) in a mixed coordinate framework with unfitted finite elements (Burman, Fernández, F., Gerosa, CMAME 2021)



Outlook

- Comparison of different numerical approaches for benchmark configuration (2d/2.5d)

Main references

- E. Burman, M.A. Fernández, S. Frei: A Nitsche-based formulation for fluid-structure interactions with contact, ESAIM M2AN 54(2), 531-564 (2020)
- S. Frei, F.M. Gerosa, E. Burman, M.A. Fernández: A mechanically consistent model for fluid-structure interactions with contact including seepage, Comp Methods Appl Mech Eng 392, 114637 (2022)
- H. von Wahl, T. Richter, S. Frei, T. Hagemeier: Falling balls in a viscous fluid with contact: Comparing numerical simulations with experimental data, Phys Fluids 33, 033304 (2021)