IVAN MEDA

1

$G(A) = \operatorname{tr} A$

$G(A+\alpha U) = \operatorname{tr}(A+\alpha U) = \operatorname{tr} A + \alpha \operatorname{tr} U$

$\frac{\partial \operatorname{tr} A}{\partial (A)}[U] = DG(A)[U] = \underline{\operatorname{tr} U}$

---

(2) $G(A) = \frac{1}{2}\left((\operatorname{tr} A)^2 - \operatorname{tr} A^2\right)$

$G(A+\alpha U) = \frac{1}{2}\left((\operatorname{tr}(A+\alpha U))^2 - \operatorname{tr}(A+\alpha U)^2\right)$

$= \frac{1}{2}\left((\operatorname{tr} A)^2 + 2\alpha \operatorname{tr} A \cdot \operatorname{tr} U + \alpha^2 (\operatorname{tr} U)^2 - \operatorname{tr} A^2 - \alpha \operatorname{tr}(AU) + \alpha \operatorname{tr}(UA) - \alpha^2 \operatorname{tr} U^2\right)$

$DG(A)[U] = \underline{\operatorname{tr} A \cdot \operatorname{tr} U - \frac{1}{2}\operatorname{tr}(AU) - \frac{1}{2}\operatorname{tr}(UA)} = \underline{\operatorname{tr} A \operatorname{tr} U - \operatorname{tr}(AU)}$

(3) $G(A) = \det A$

$G(A+\alpha U) = \det(A+\alpha U) = \det A \cdot \det(I + \underbrace{\alpha A^{-1}U}_{S}) = \det A\,(\underbrace{\alpha A^{-1}U}_{S} + I)$

$\left[\det(\lambda I + S) = -\lambda^3 + \lambda^2 \operatorname{tr} S - \lambda \frac{1}{2}\left((\operatorname{tr} S)^2 - (\operatorname{tr} S^2)\right) + \det S \right]$

$= \det A \left(\det(\alpha A^{-1}U) + \frac{1}{2}\left((\operatorname{tr} \alpha A^{-1}U)^2 - (\operatorname{tr} \alpha^2 A^{-2}U^2)\right) + \operatorname{tr} \alpha A^{-1}U + 1\right)$

~~$DG(A)[U] = \lim_{\alpha \to 0} \frac{1}{\alpha} \det A\,(I - \alpha \det(A^{-1}U) +$~~

$DG(A)[U] = \lim_{\alpha \to 0} \frac{1}{\alpha} \cdot \left[\det A \cdot \left(\det(\alpha A^{-1}U) + \alpha^2(\ldots) + 1\right) - \det A\right]$

$= \lim_{\alpha \to 0} \frac{1}{\alpha}\left[\det A \cdot \left(1 - 1 + \alpha^3 \det(A^{-1}U) + \frac{\alpha^2}{2}(\ldots) + \alpha \cdot \operatorname{tr} A^{-1}U\right)\right] =$

$= \underline{\det A \cdot \operatorname{tr}(A^{-1}U)}$