1. Let $\mathbf{v} =_{\text{def}} \frac{\mathbf{r}}{|\mathbf{r}|^n}$, $n \in \mathbb{N}$, be a vector field in $\mathbb{R}^3 \setminus \{\mathbf{0}\}$, where $\mathbf{r} =_{\text{def}} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^\top$ denotes the position vector and $|\cdot|$ denotes the standard Euclidean norm. Find by direct computation $\operatorname{rot}\operatorname{rot}\mathbf{v}$, $\Delta\mathbf{v}$ and $\nabla(\operatorname{div}\mathbf{v})$, and verify that

$$\operatorname{rot}\operatorname{rot}\mathbf{v} = \nabla\operatorname{div}\mathbf{v} - \Delta\mathbf{v}.$$

Recall that $\Delta\mathbf{v} =_{\text{def}} \operatorname{div}(\nabla\mathbf{v})$. It might be convenient to first find formulae for $\operatorname{div}\mathbf{r}$, $\nabla|\mathbf{r}|$ and so on, and then to proceed using the identities of the type $\operatorname{div}(\varphi\mathbf{u}) = \mathbf{u} \bullet \nabla\varphi + \varphi\operatorname{div}\mathbf{u}$ and so on. (See the list presented during the last tutorial.)

2. Show that

$$\begin{aligned}
\frac{\partial^2 \mathrm{I}_1(\mathbb{A})}{\partial\mathbb{A}^2}[\mathbb{B},\mathbb{C}] &= 0, \\
\frac{\partial^2 \mathrm{I}_2(\mathbb{A})}{\partial\mathbb{A}^2}[\mathbb{B},\mathbb{C}] &= (\operatorname{Tr}\mathbb{C})(\operatorname{Tr}\mathbb{B}) + \operatorname{Tr}(\mathbb{C}\mathbb{B}), \\
\frac{\partial^2 \mathrm{I}_3(\mathbb{A})}{\partial\mathbb{A}^2}[\mathbb{B},\mathbb{C}] &= (\det\mathbb{A})\left(\operatorname{Tr}\left(\mathbb{A}^{-1}\mathbb{B}\right)\operatorname{Tr}\left(\mathbb{A}^{-1}\mathbb{C}\right) - \operatorname{Tr}\left(\mathbb{A}^{-1}\mathbb{B}\mathbb{A}^{-1}\mathbb{C}\right)\right),
\end{aligned}$$

where $\mathrm{I}_1(\mathbb{A})$, $\mathrm{I}_2(\mathbb{A})$ and $\mathrm{I}_3(\mathbb{A})$ denote the principal invariants of matrix $\mathbb{A}$, that is

$$\begin{aligned}
\mathrm{I}_1(\mathbb{A}) &=_{\text{def}} \operatorname{Tr}\mathbb{A}, \\
\mathrm{I}_2(\mathbb{A}) &=_{\text{def}} \frac{1}{2}\left((\operatorname{Tr}\mathbb{A})^2 - \operatorname{Tr}\left(\mathbb{A}^2\right)\right), \\
\mathrm{I}_3(\mathbb{A}) &=_{\text{def}} \det\mathbb{A}.
\end{aligned}$$