1. Prove the following lemma. Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with a smooth boundary. Let $\mathbf{v}$ be a smooth vector field that vanishes on the boundary $\mathbf{v}|_{\partial\Omega} = 0$. Then
$$2 \int_{\Omega} \mathbb{D} : \mathbb{D} \, \mathrm{dV} = \int_{\Omega} \nabla \mathbf{v} : \nabla \mathbf{v} \, \mathrm{dV} + \int_{\Omega} (\operatorname{div} \mathbf{v})^2 \, \mathrm{dV},$$
where $\mathbb{D}$ denotes the symmetric part of the gradient of $\mathbf{v}$, $\mathbb{D} =_{\mathrm{def}} \frac{1}{2} \left(\nabla \mathbf{v} + (\nabla \mathbf{v})^{\top} \right)$.

2. Prove the following lemma. Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with a smooth boundary, and let $\mathbf{v}$ be a smooth vector field, then
$$\int_{\Omega} \operatorname{rot} \mathbf{v} \, \mathrm{dV} = - \int_{\partial\Omega} \mathbf{v} \times \mathbf{n} \, \mathrm{dS}.$$

3. Show that if $\Omega \subset \mathbb{R}^3$ is a bounded domain with a sufficiently smooth boundary, then
$$\int_{\partial\Omega} \mathrm{d}\mathbf{S} = \mathbf{0}.$$