

1. Let \mathbf{u} be an Eulerian vector field. Is it true that $\frac{d}{dt}(\operatorname{div} \mathbf{v}) = \operatorname{div} \left(\frac{d\mathbf{v}}{dt} \right)$, where $\frac{d}{dt}$ denotes the material time derivative? If not, what is the difference between the two expressions?
2. Consider the deformation shown in Figure 1. Assume that the deformation is described using the cylindrical coordinate system in the current configuration, that is function $\chi(\mathbf{X}, t)$ is given by the formulae

$$\begin{aligned} r &= f(X, Y), \\ \varphi &= g(X, Y), \\ z &= Z, \end{aligned}$$

while the relation between the Cartesian coordinates in the current configuration $[x, y, z]$ and the cylindrical coordinates $[r, \varphi, z]$ in the current configuration reads

$$\begin{aligned} x &= r \cos \varphi, \\ y &= r \sin \varphi, \\ z &= z. \end{aligned}$$

Find a formula for the deformation gradient \mathbb{F} provided that we want to use Cartesian coordinate system in the reference configuration and the cylindrical coordinate system in the current configuration.

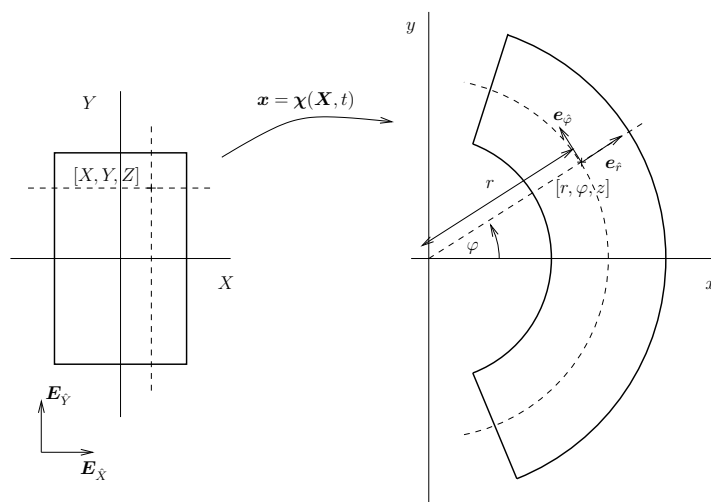


Figure 1: Problem geometry.