1. Consider linearised homogeneous isotropic elastic solid, that is a continuous medium where the Cauchy stress tensor is given by the formula
$$\mathbb{T} = \lambda \left(\operatorname{Tr} \varepsilon\right) \mathbb{I} + 2\mu\varepsilon,$$
where $\varepsilon =_{\mathrm{def}} \frac{1}{2}\left(\nabla \mathbf{U} + \left(\nabla \mathbf{U}\right)^{\top}\right)$ is the linearised strain. Show that the (linearised) governing equations in $\mathbb{R}^3$ admit, if there are no specific body forces, a solution in the form of a wave
$$\mathbf{U} = \mathbf{A} \sin\left(\mathbf{K} \bullet \mathbf{X} - \omega t\right), \tag{1}$$
where $\mathbf{A}$ denotes the amplitude of the wave, vector $\mathbf{K}$ denotes the wave vector that determines the direction of the propagation of the wave and the spatial frequency of the wave, and ω is the angular frequency. (The speed of propagation of the wave is given by the formula $c =_{\mathrm{def}} \frac{\omega}{K}$, where $K =_{\mathrm{def}} |\mathbf{K}|$ is called the wavenumber.)
In particular, show that $\mathbf{A} \sin\left(\mathbf{K} \bullet \mathbf{X} - \omega t\right)$ is a solution to the (linearised) governing equations provided that *either* $\mathbf{A}$ is parallel to $\mathbf{K}$ and the speed of propagation is $c_{\parallel} = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ *or* $\mathbf{A}$ is perpendicular to $\mathbf{K}$ and the speed of propagation is $c_{\perp} = \sqrt{\frac{\mu}{\rho}}$.

Having solved the problem, you can enjoy reading the following paper on seismic waves, *František Gallovič: Pár sekund k dobru: Systémy včasného varování před zemětřeseními (in Czech), Vesmír 91, 510-512, 2012.*