1. Show that

$$\left. \frac{\mathrm{d}\mathbb{U}_t(\boldsymbol{x}, \tau)}{\mathrm{d}\tau} \right|_{\tau = t} = \mathbb{D}(\boldsymbol{x}, t),$$

where $\mathbb{U}_t(\boldsymbol{x}, \tau)$ denotes the relative stretch tensor, that is the symmetric positive definite matrix from the polar decomposition of the relative deformation gradient $\mathbb{F}_t(\boldsymbol{x}, \tau)$,

$$\mathbb{F}_t(\boldsymbol{x}, \tau) = \mathbb{R}_t(\boldsymbol{x}, \tau)\, \mathbb{U}_t(\boldsymbol{x}, \tau),$$

and $\mathbb{D}(\boldsymbol{x}, t)$ and $\mathbb{W}(\boldsymbol{x}, t)$ denote the symmetric and skew-symmetric part of the velocity gradient. Recall that we already know that

$$\left. \frac{\mathrm{d}\mathbb{F}_t(\boldsymbol{x}, \tau)}{\mathrm{d}\tau} \right|_{\tau = t} = \mathbb{L}(\boldsymbol{x}, t),$$

$$\mathbb{L}(\boldsymbol{x}, t) = \mathbb{D}(\boldsymbol{x}, t) + \mathbb{W}(\boldsymbol{x}, t),$$

$$\left. \mathbb{F}_t(\boldsymbol{x}, \tau) \right|_{\tau = t} = \mathbb{I},$$

where $\mathbb{L}(\boldsymbol{x}, t)$ denotes the velocity gradient.

2. Let $\boldsymbol{v}$ denote the Eulerian velocity field. Show that

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\partial \boldsymbol{v}}{\partial t} + (\operatorname{rot} \boldsymbol{v}) \times \boldsymbol{v} + \nabla \left(\frac{1}{2} \boldsymbol{v} \bullet \boldsymbol{v} \right),$$

where $\frac{\mathrm{d}}{\mathrm{d}t}$ is the material time derivative. Further, show that

$$\dot{\overline{\operatorname{rot} \boldsymbol{v}}} = \operatorname{rot} \dot{\boldsymbol{v}} + ((\operatorname{rot} \boldsymbol{v}) \bullet \nabla)\, \boldsymbol{v} - (\operatorname{rot} \boldsymbol{v}) \operatorname{div} \boldsymbol{v}.$$